

Polarization studies in the Muon Collider and the neutrino factory

Rajendran Raja

Fermilab

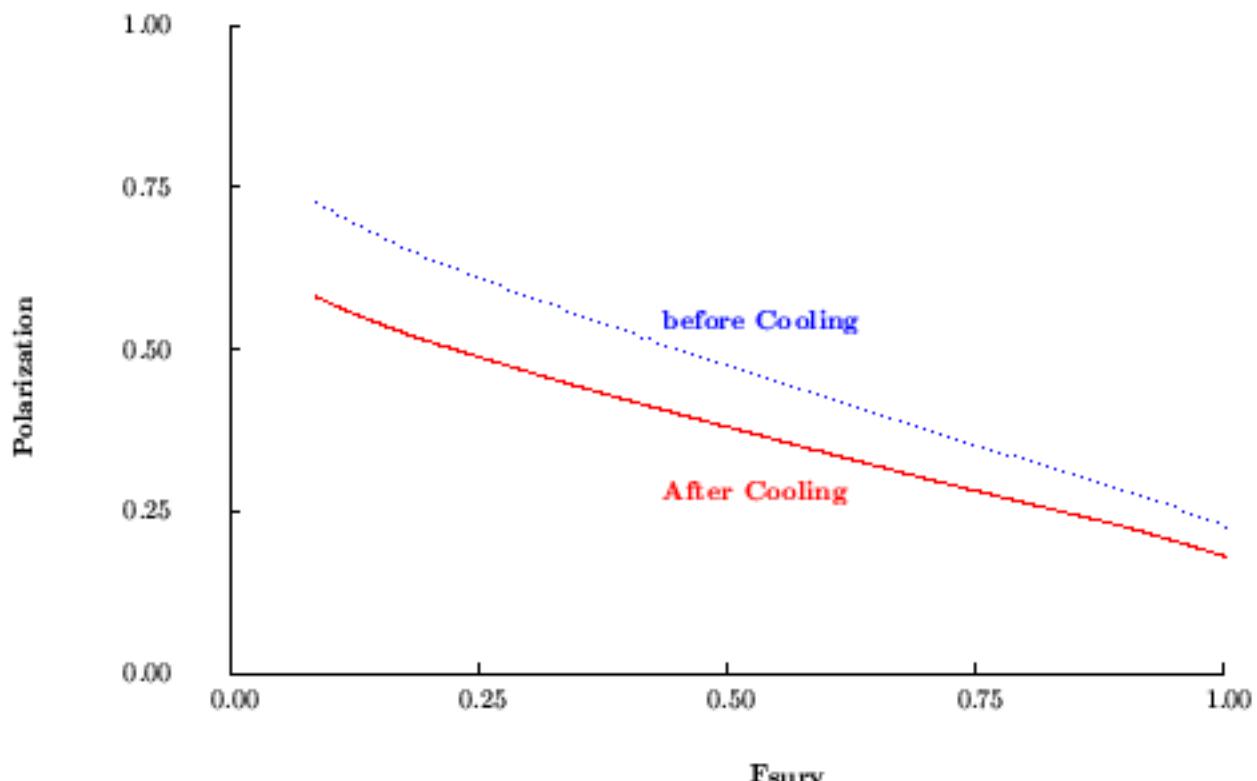
HIMUS99, KEK, JAPAN

Summary of talk

- Polarized muons
 - » 25% polarizations are easy to achieve
- Measurement of energy scale using g-2 precession
- Energy of muon neutrinos and electron anti-neutrinos (from μ^- decay) change with polarization.
- How well can we determine δm^2 $\sin^2 2\theta$ from muon disappearance measurements? (Barger, Geer, Raja Whisnant -FNAL-PUB-99-341-T)
- If we do not take into account polarization, how do we affect these measurements?
- Ongoing work and conclusions.

Muon Collider Physics

- Polarization of muons will play a crucial role in many physics areas.
- Both charges polarizable.



Calibrating the energy of the collider to 1E-6

Bargmann-Michel-Telegdi Equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

$$\vec{\Omega} = -\frac{e}{m\gamma} ((1+a\gamma)\vec{B}_\perp + (1+a)\vec{B}_\parallel - (a\gamma + \frac{\gamma}{1+\gamma})\vec{\beta} \times \frac{\vec{E}}{c})$$

$$\vec{\Omega} = \vec{\Omega}_{\text{osc}}(1+a\gamma)$$

$$a = (g-2)/2$$

$\vec{B}_\perp, \vec{B}_\parallel$ are the components of magnetic field perpendicular and parallel particle direction

This equation controls the evolution of the spin vector \vec{S} . Polarization is the average of the spin vectors over the muon ensemble. Per revolution spin rotates by $a\gamma 2\pi$ radians more than momentum

Method described in

R.Raja and A. Tollestrup, Phys. Rev. D58(1998)013005

Definitions

In the muon rest frame, E is the energy of the electron. Its fractional energy expressed in terms of the maximum energy ($m_\mu/2$) is x . N is the number of muon decays. θ is the angle of the electron in the muon center of mass w.r.t muon direction. $\langle E \rangle$ is the average electron energy and $\langle PL \rangle$ is the average longitudinal electron momentum in the muon rest frame. P is the z component of the muon polarization along the muon direction. \hat{P} is charge* P of the muon.

Electron energy distribution

$$x = 2E/m_\mu$$

$$\frac{d^2N}{dxd\cos\theta} = N(x^2(3-2x) - \hat{P}x^2(1-2x)\cos\theta)$$

$$\langle E \rangle = \frac{m_\mu}{2N} \iint x \frac{d^2N}{dxd\cos\theta} dx d\cos\theta = \frac{7}{10} \frac{m_\mu}{2}$$

$$\langle P_L \rangle = \frac{m_\mu}{2N} \iint x \cos\theta \frac{d^2N}{dxd\cos\theta} dx d\cos\theta = \frac{\hat{P}}{10} \frac{m_\mu}{2}$$

Muon neutrinos have identical distribution to electrons.

Electron anti-neutrinos have the following distribution.

$$\frac{d^2N}{dxd\cos\theta} = 6(x^2(1-x) - \hat{P}x^2(1-x)\cos\theta)$$

$$\langle E \rangle = \frac{6}{20} m_\mu$$

$$\langle P_L \rangle = -\frac{\hat{P}}{10} m_\mu$$

Decay distributions

$$\langle E_{lab} \rangle = \frac{7}{20} E_\mu (1 + \frac{\beta}{7} \hat{P})$$

$$E(t) = N e^{-\alpha t} (\frac{7}{20} E_\mu (1 + \frac{\beta}{7} (\hat{P} \cos \omega t + \phi)))$$

$$\omega = \gamma \frac{g - 2}{2} 2\pi$$

$$\alpha = \frac{t_{circ}}{\eta_{life}} = \frac{2\pi m_\mu}{0.3 B c t_{life}}$$

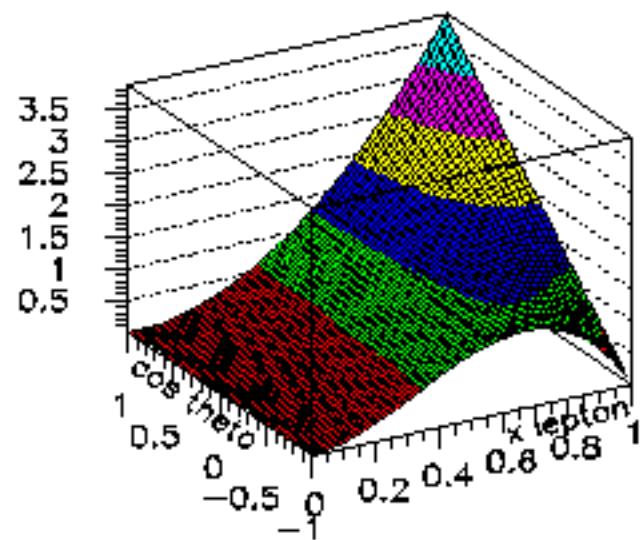
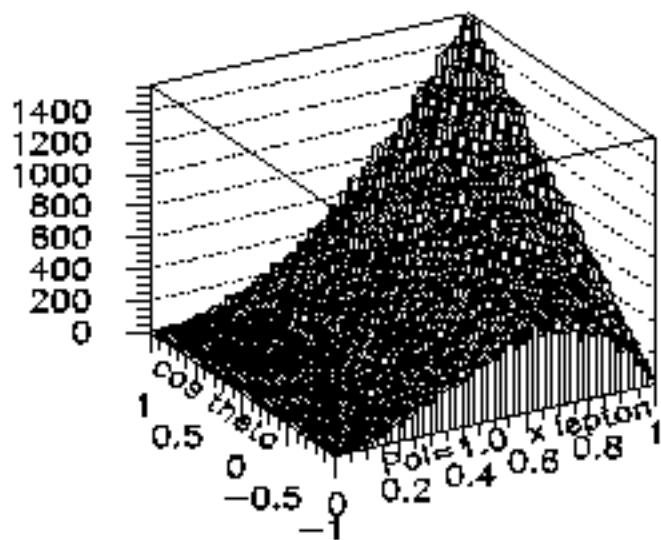
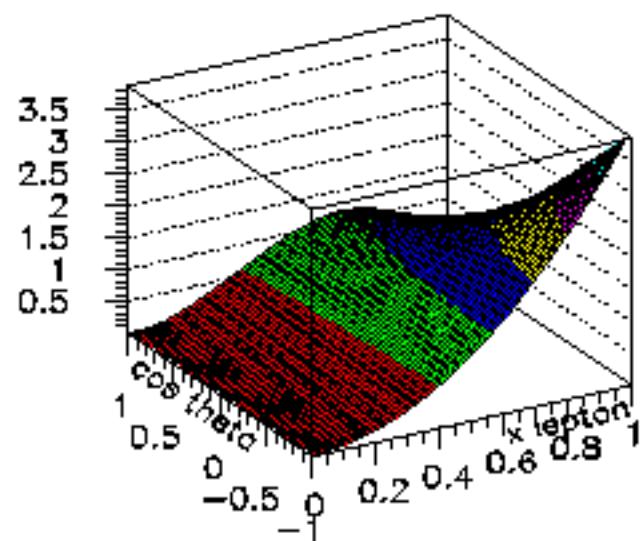
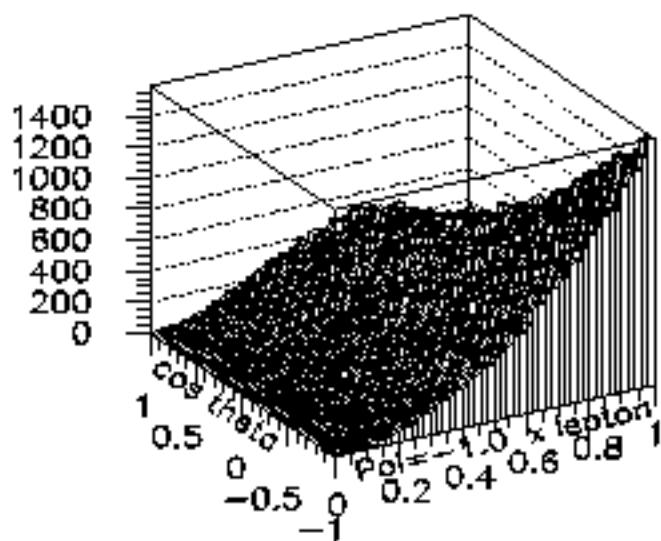
$$f(t) = A e^{-Bt} (C \cos(D + Et) + F)$$

$\langle E_{lab} \rangle$ is the average electron energy in lab. $E(t)$ is the total electron energy during turn t . Determine ω to get γ . γ information also present in α .

$f(t)$ is the fitting function. MINUIT used to fit and extract information.

Electron energy and angle distributions in muon rest frame
Polarization = -1.0 and 1.0

19/08/97 14.24

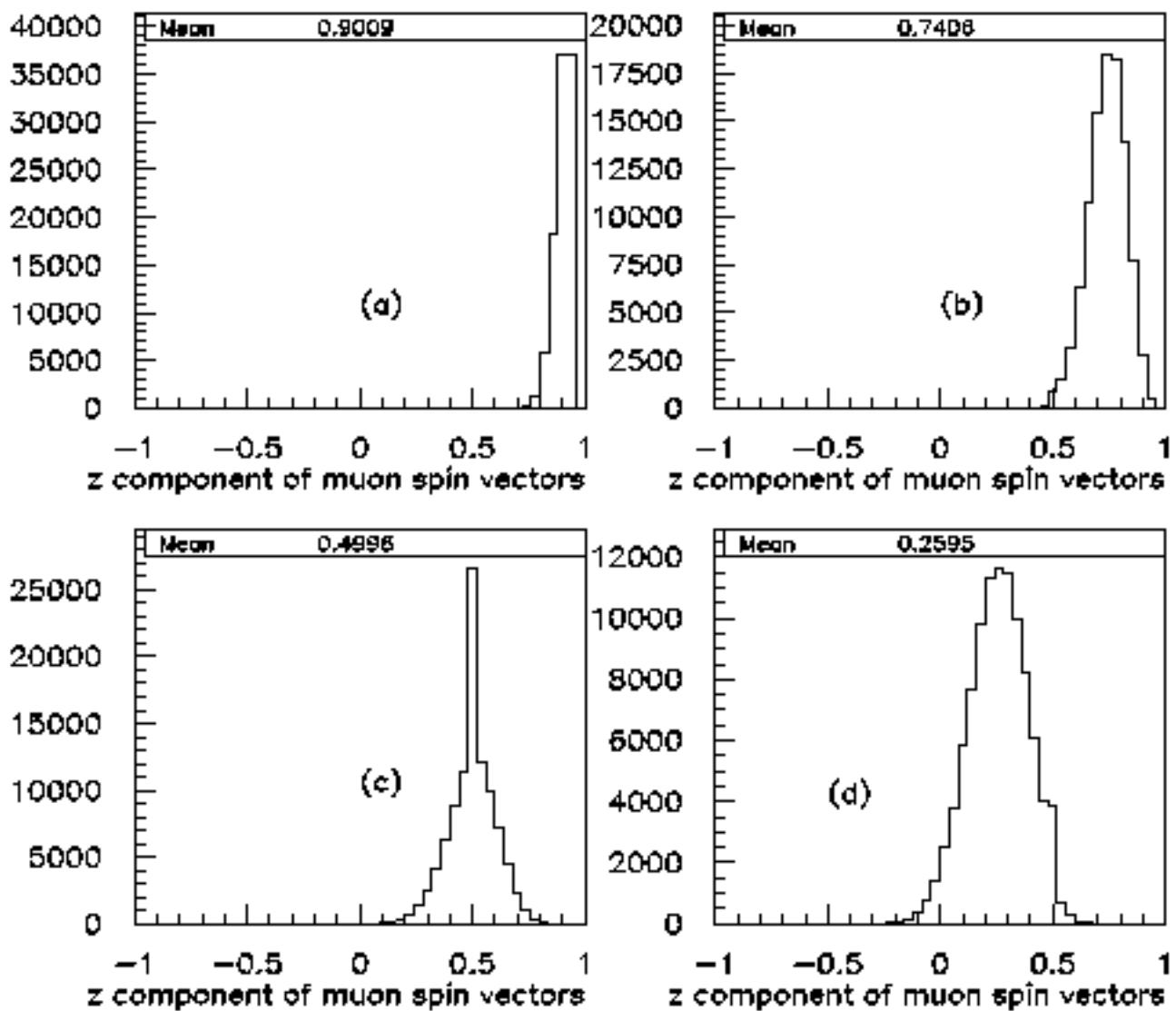


Variation in spin vectors

The electron energy sampled depends only on \hat{P} which the average of the z component of the individual spin vectors. If all the muons had the same momentum, then the variation of the individual spin vectors is unimportant. Only the average matters. However, $\delta p/p$ is non-zero and the individual spins precess slightly differently turn to turn and a dilution in polarization results. We generate \hat{P} as a binomial distribution about a given average.

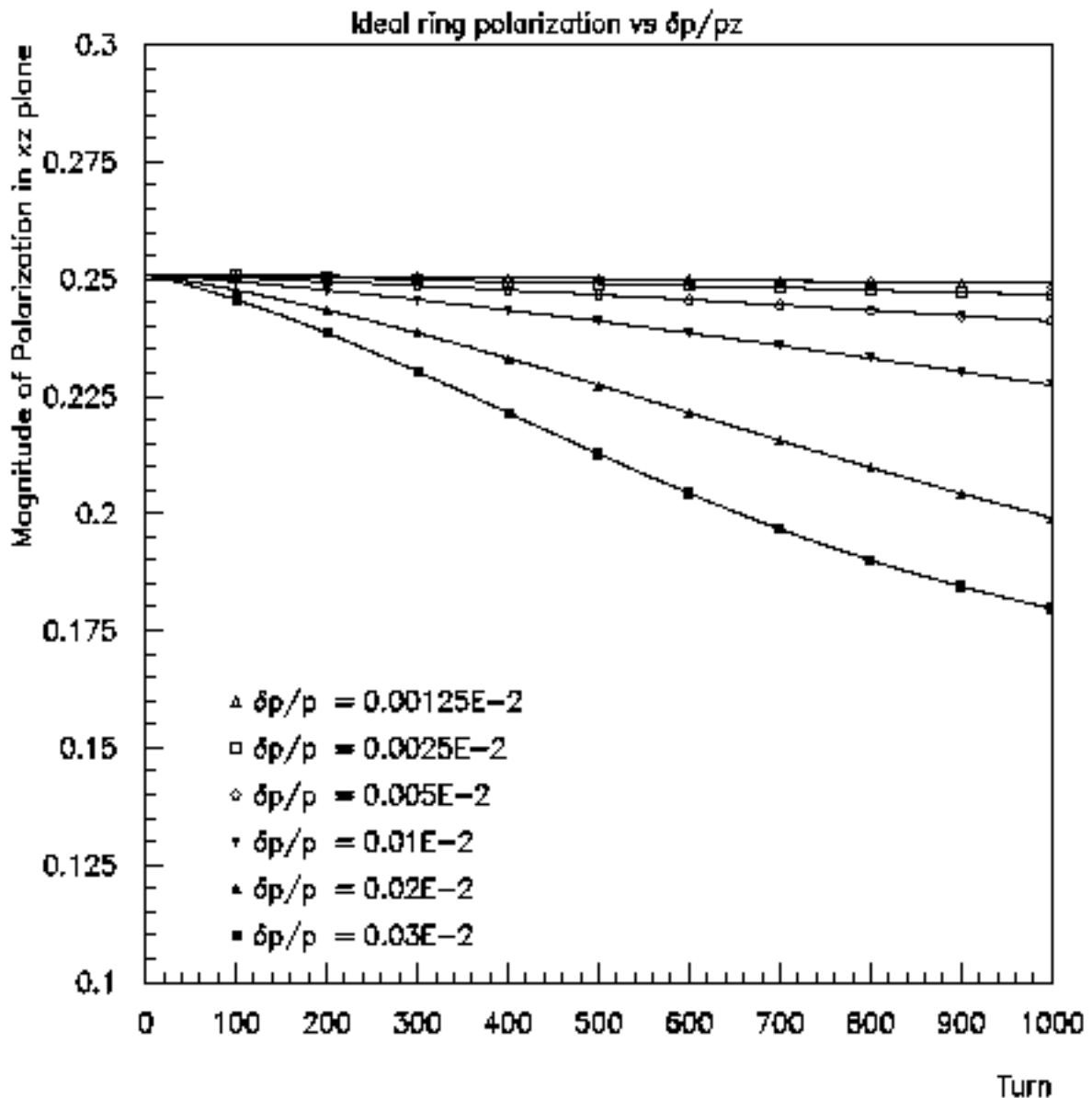
Muon Spin vector generation

05/09/97 10.28



Polarization vs turn for various $\delta p/p$

05/11/97 10.57



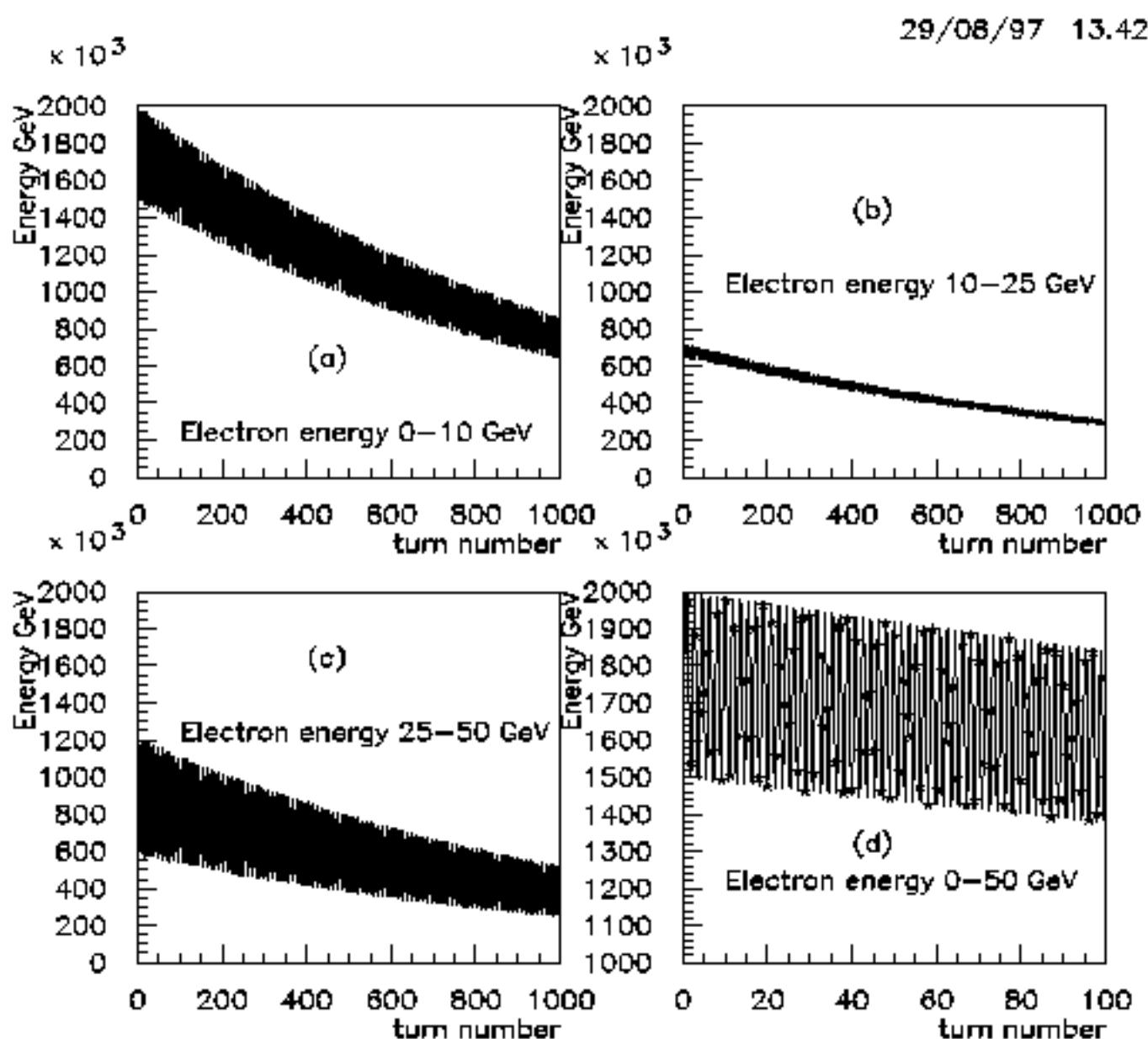
Idealized collider ring model

In order to simulate the decays, we assume an idealized planar collider ring made up of 4.0 Tesla average bending field. This results in the following parameters.

Muon Energy	= 50 GeV
Lorentz factor γ	= 473.22
Spin precession per turn	= 3.4667 radians
Magnetic field	= 4 Tesla
Ring Radius	= 41.667 meters
Circulation time	= 0.873E-6 sec
Dilated muon life time	= 0.104E-2 sec
Turn by turn decay constant α	= 0.8399E-3

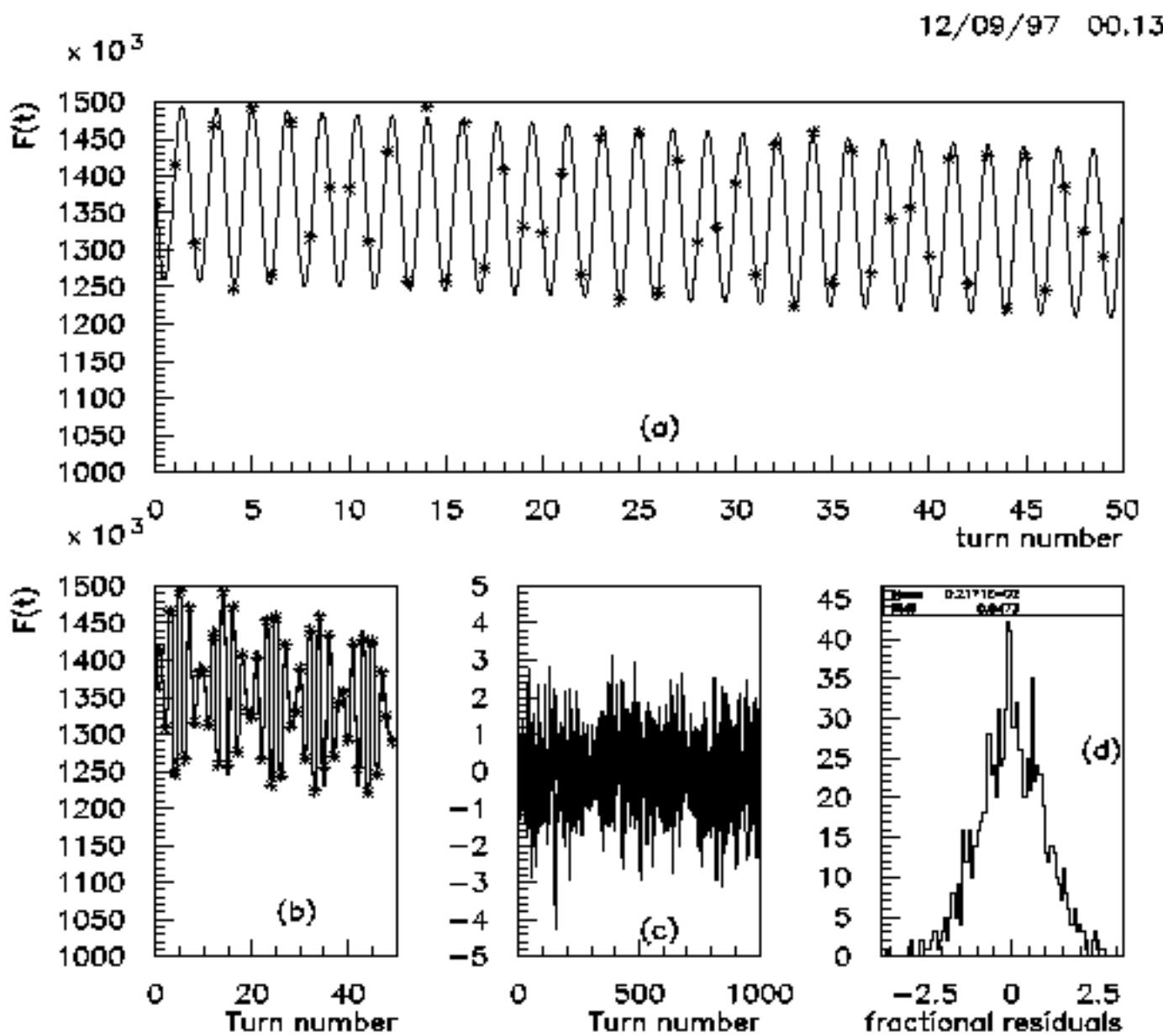
.

Electron lab energy spectrum Pol=1.0, 100K decays



Fit to 50 GeV μ , P=0.26

$\delta p/p = 0.03E-2$



*$\delta\gamma/\gamma$ vs measurement error
and Polarization $\delta p/p = 0.03E-2$*

12/09/97 00.20

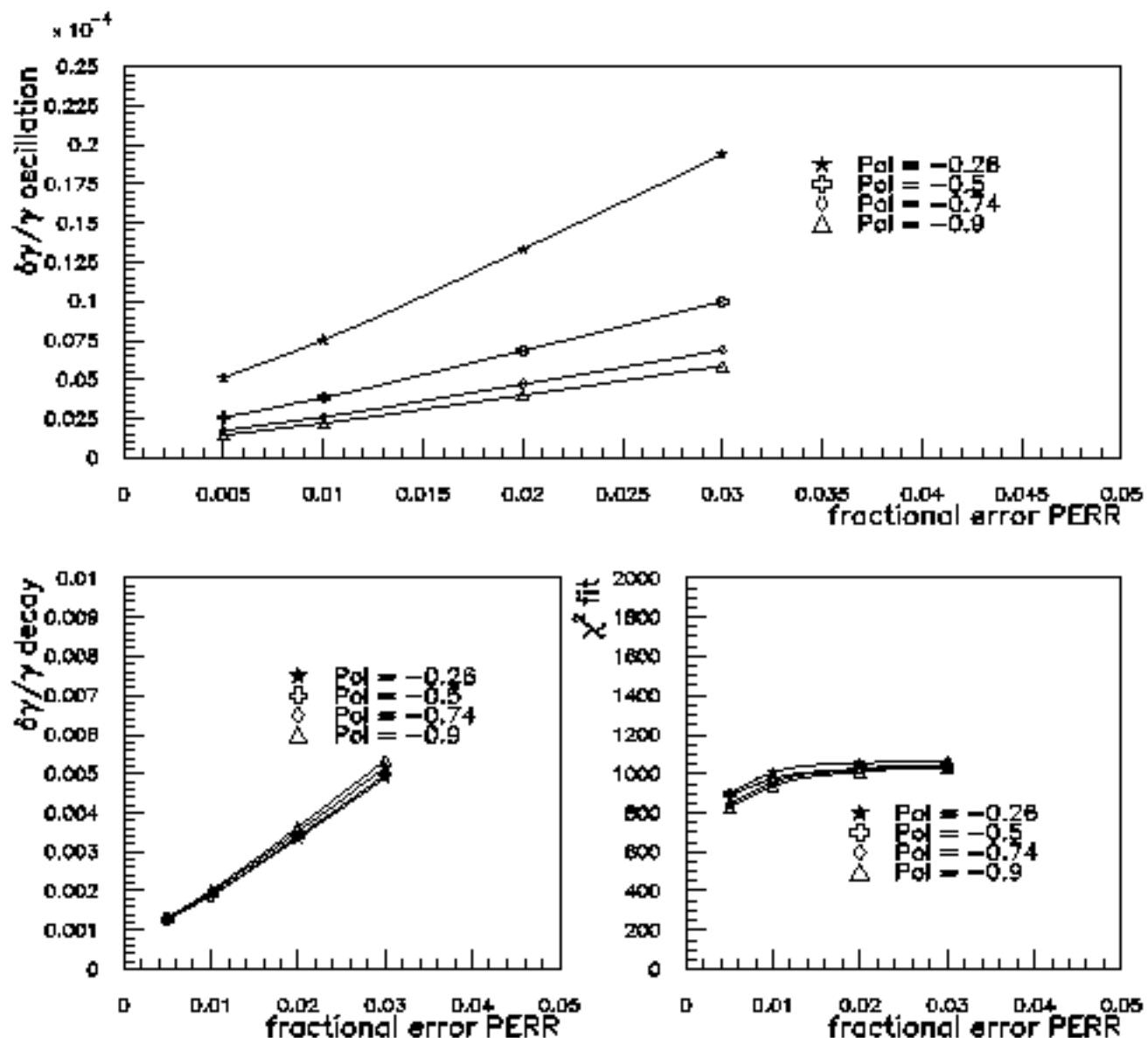


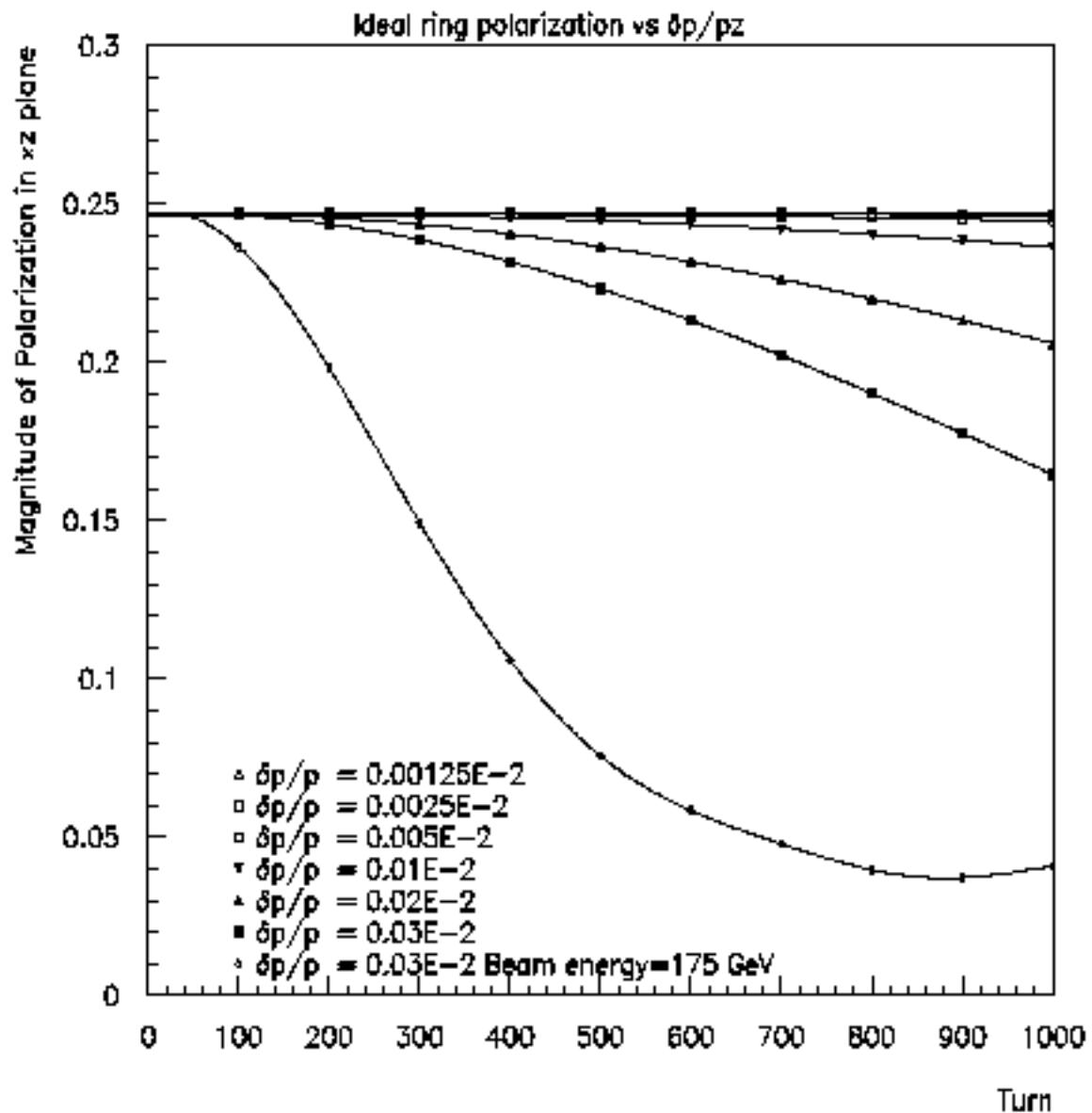
Table of fit parameters

\hat{P}	PERR	Number of electrons sampled	$\delta\gamma/\gamma_{oscillations}$	$\delta\gamma/\gamma_{decay}$	χ^2 for NDF=1000
-0.90	0.50E-02	41261	0.14568E-05	0.13227E-02	824.
-0.90	0.10E-01	10315	0.22147E-05	0.20124E-02	936.
-0.90	0.20E-01	2579	0.39999E-05	0.36398E-02	1009.
-0.90	0.30E-01	1146	0.58659E-05	0.53457E-02	1030.
-0.74	0.50E-02	41261	0.17418E-05	0.13019E-02	843.
-0.74	0.10E-01	10315	0.26183E-05	0.19591E-02	954.
-0.74	0.20E-01	2579	0.46981E-05	0.35229E-02	1021.
-0.74	0.30E-01	1146	0.68765E-05	0.51672E-02	1039.
-0.50	0.50E-02	41261	0.25903E-05	0.12813E-02	888.
-0.50	0.10E-01	10315	0.38407E-05	0.19029E-02	973.
-0.50	0.20E-01	2579	0.68338E-05	0.33972E-02	1026.
-0.50	0.30E-01	1146	0.99744E-05	0.49749E-02	1041.
-0.26	0.50E-02	41261	0.51242E-05	0.12688E-02	898.
-0.26	0.10E-01	10315	0.75317E-05	0.18791E-02	1004.
-0.26	0.20E-01	2579	0.13324E-04	0.33447E-02	1053.
-0.26	0.30E-01	1146	0.19380E-04	0.48950E-02	1061.

TABLE I. Results of fits for $\delta\gamma/\gamma$ as a function of polarization \hat{P} and noise PERR. Also shown is the χ^2 of the fit for 1000 turns.

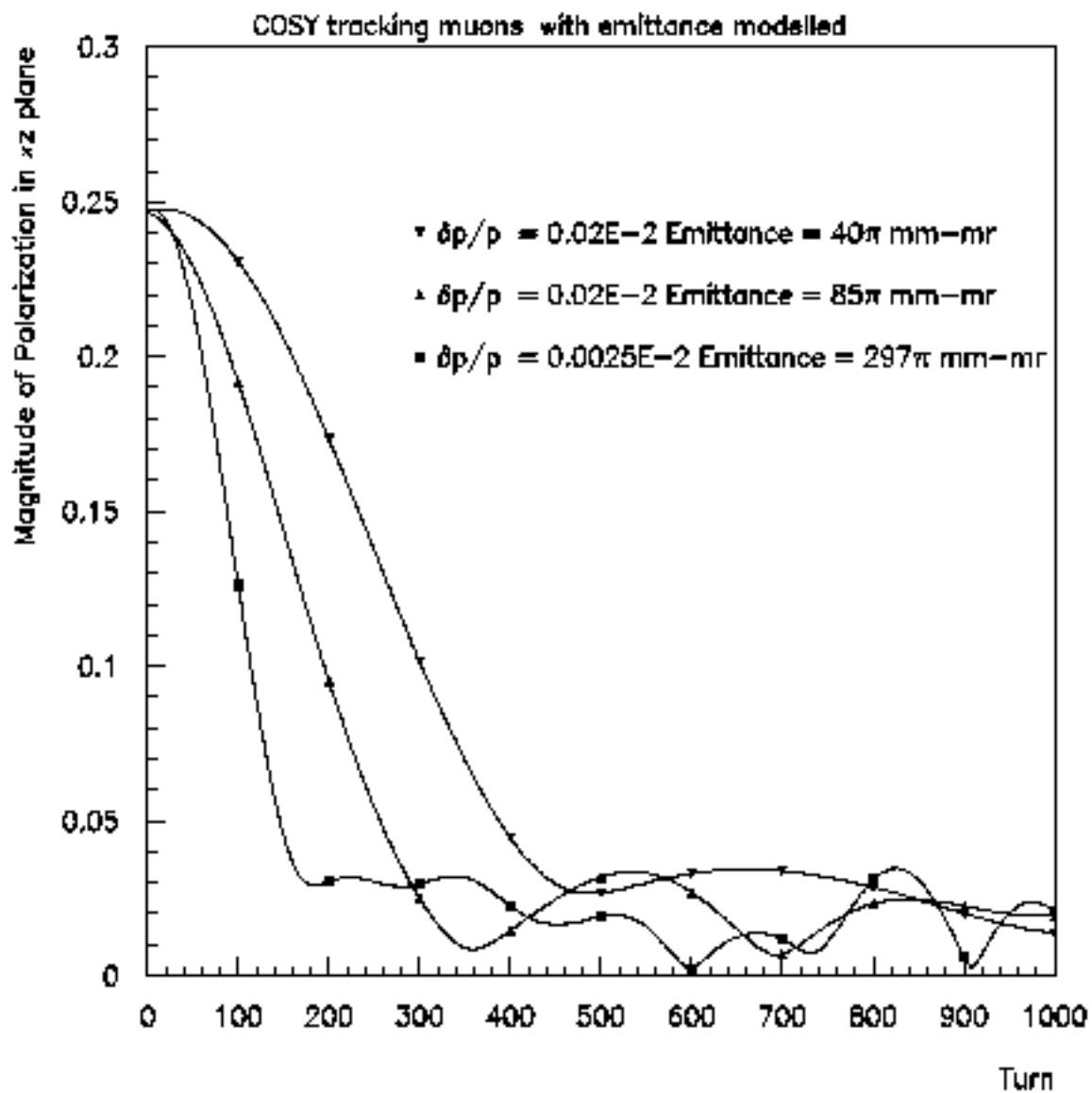
Polarization vs turn for various $\delta p/p$

23/11/97 19.40

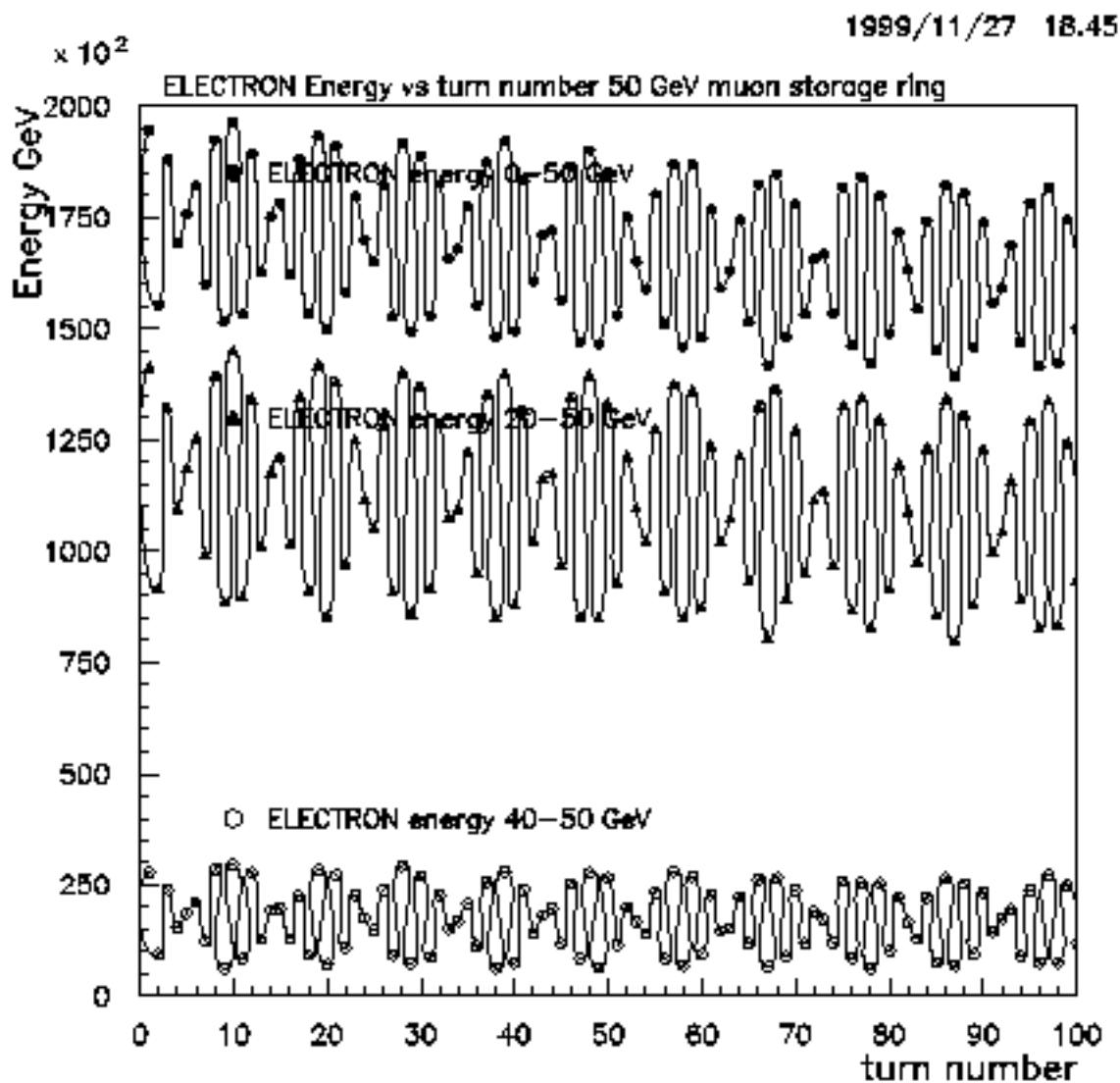


Polarization of 1000 muons vs turn number in COSY

22/11/97 18.43

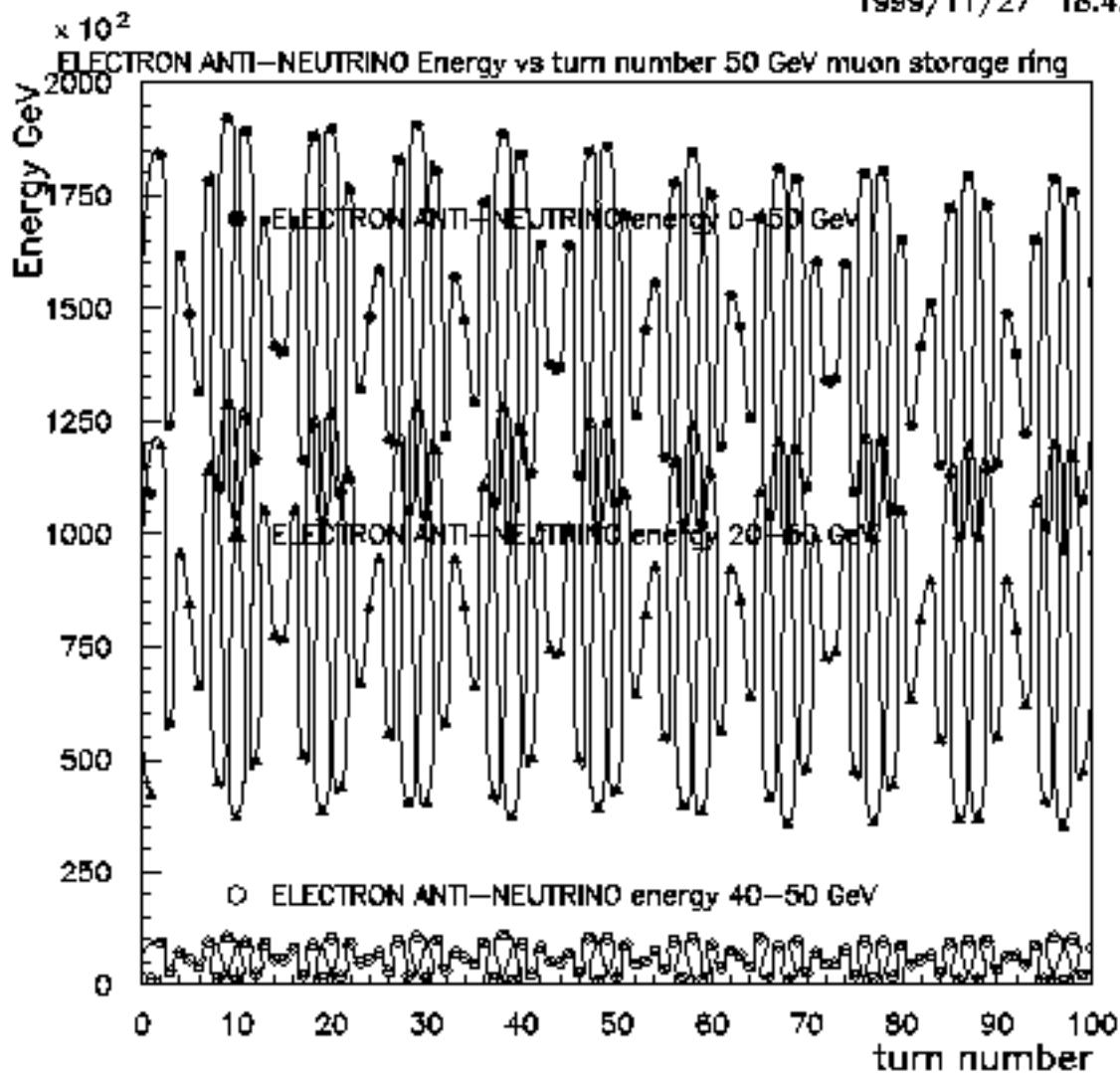


Energy distribution of electrons



Energy distribution of electron anti-neutrinos

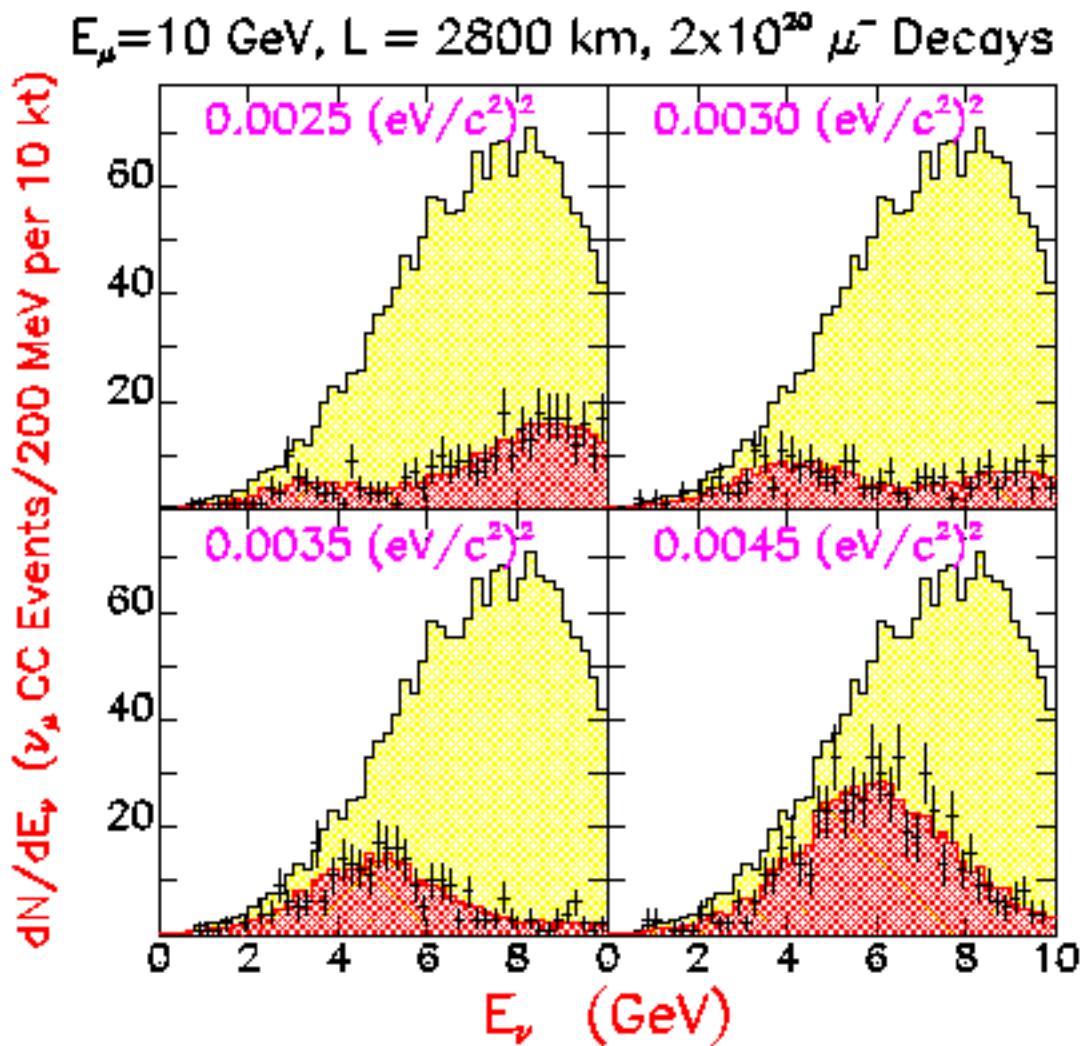
1999/11/27 18.43



- Bow tie vs simple ring? Can polarization survive a 100 turns?

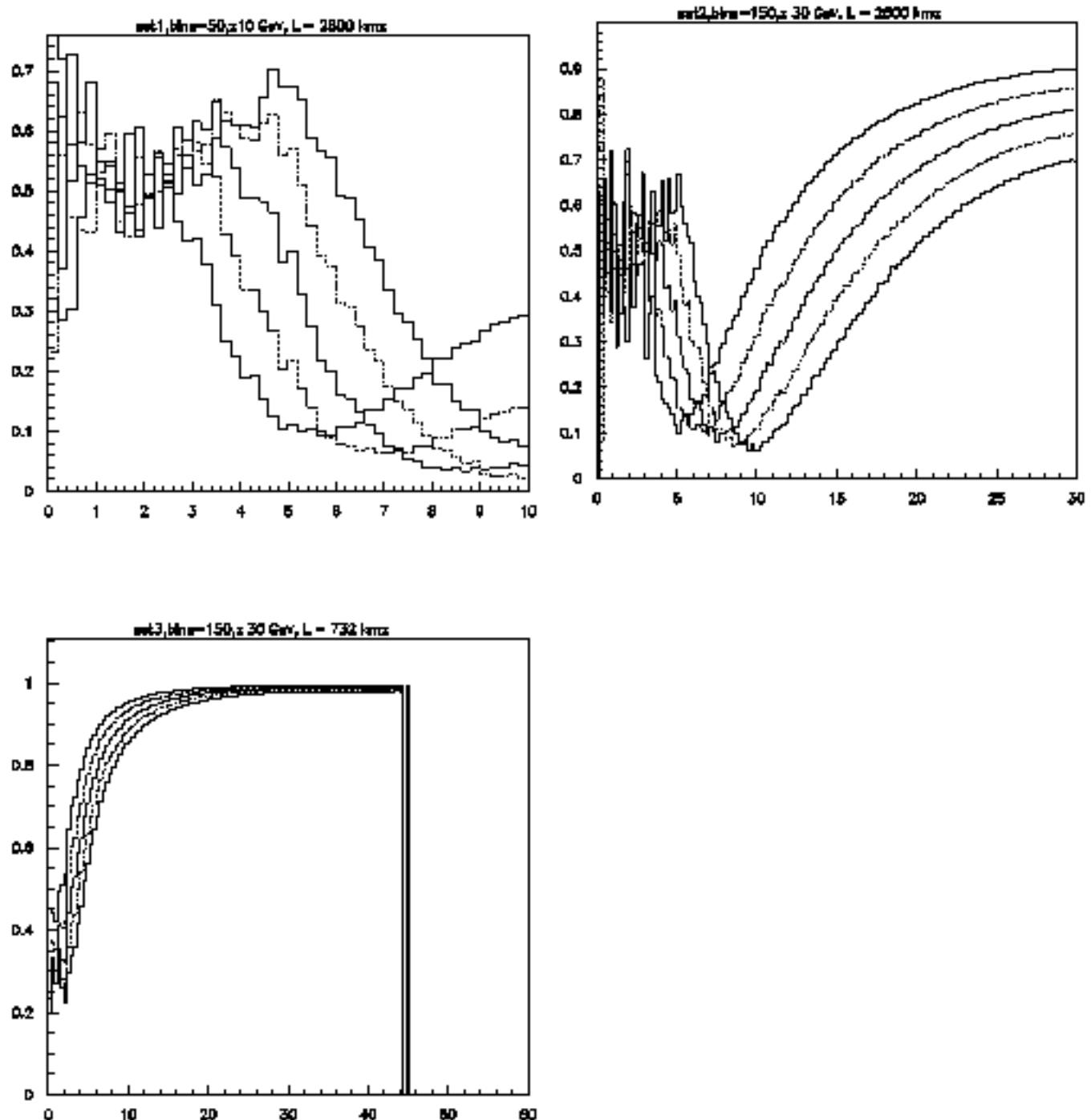
Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $\nu_\mu \rightarrow \nu_\mu$

- Barger, Geer, Raja, Whisnant Paper FNAL-PUB-99-341-T



- Interpolate using divided differences in $\delta m^2, \sin^2 2\theta$ space.

Ratio of survival/non-oscillation $\nu_\mu \rightarrow \nu_\mu$



Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $V_\mu \rightarrow V_\mu$

$$\frac{dN}{dE_{obs}} = \int dE_{true} P(E_{obs} | E_{true}) \frac{dN}{dE_{true}} (1 - \cos^4 \theta_{31} \sin^2 2\theta_{32} \sin^2 (1.267 \partial m_{32}^2 L / E_{true}))$$

$$\begin{aligned} \frac{dN}{dE_{obs}} &= \int dE_{true} P(E_{obs} | E_{true}) \frac{dN}{dE_{true}} \\ &- \cos^4 \theta_{31} \sin^2 2\theta_{32} \int dE_{true} \frac{dN}{dE_{true}} P(E_{obs} | E_{true}) \sin^2 (1.267 \partial m_{32}^2 L / E_{true}) \end{aligned}$$

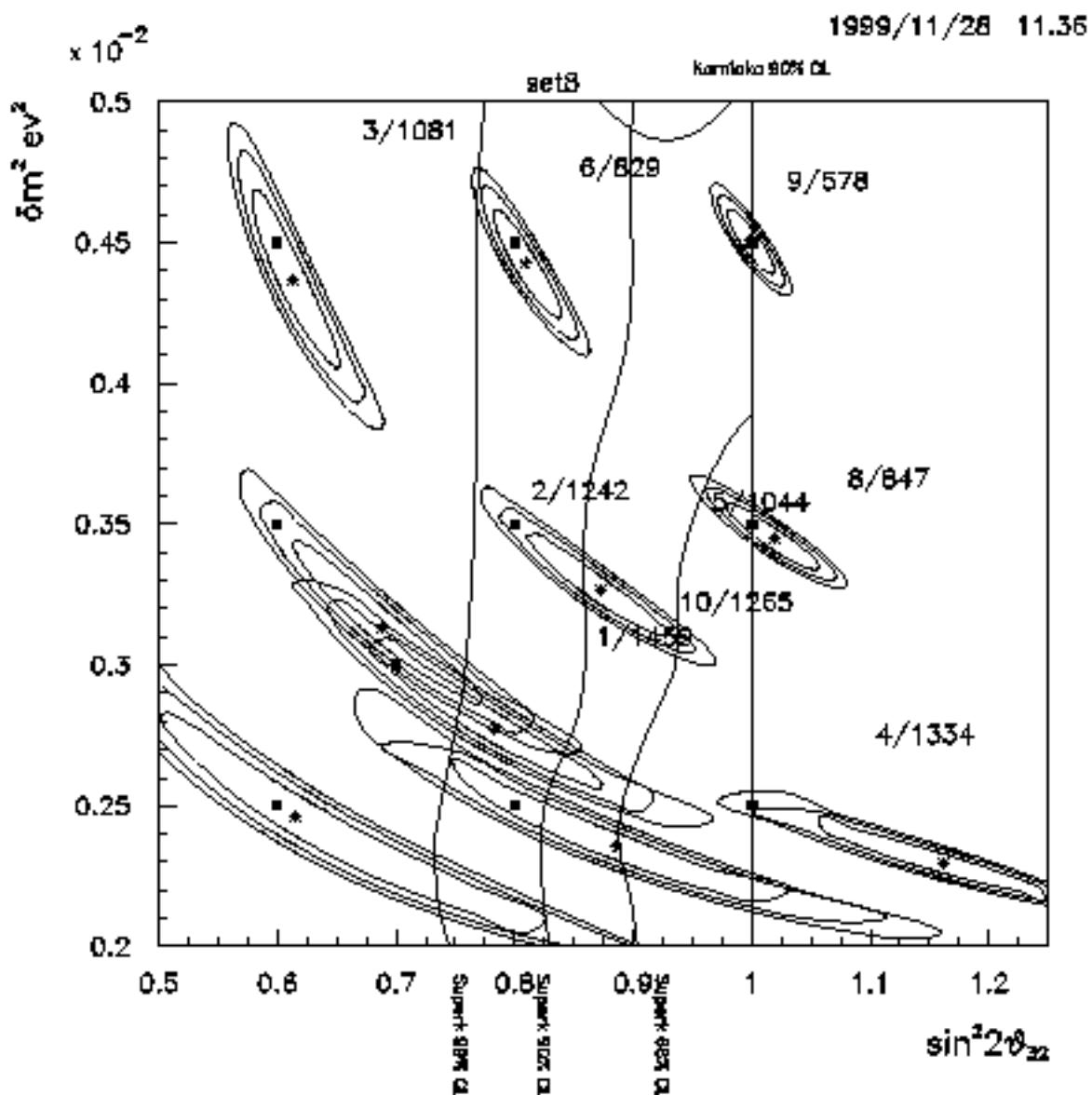
$$\Lambda = - \sum_{i=1}^{i=nevents} \log_e P^{theor}_i + \frac{(N_{exp} - N_{theor})^2}{2N_{exp}}$$

Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $V_{\mu} \rightarrow V_{\mu}$

Parameter	Value		
Muon flux	2×10^{21} decays/year		
Detector fiducial exposure	10 kiloton-years		
δm_{32}^2 values	$0.0025, 0.0030, 0.0035, 0.0040, 0.0045$ eV^2/c^4		
δm_{21}^2	$5.0 \times 10^{-5} eV^2/c^4$		
δm_{31}^2	$3.5 \times 10^{-5} eV^2/c^4$		
$\sin^2(2\theta_{12})$	0.18		
$\sin^2(2\theta_{23})$	1.0		
$\sin^2(2\theta_{31})$	0.04		
δ (CP violating phase)	0.0		
Set number	Muon momentum GeV/c	Distance ring to detector (km)	Fitting energy range GeV
Set 1	10.0	2800.	0-10
Set 2	30.0	2800.	0-12
Set 3	30.0	732.	0-12
Set 4	30.0	7332.	0-25
Set 5	50.0	2800.	0-10
Set 6	50.0	732.	0-12
Set 7	50.0	7332.	0-28
Set 8	10.0	732.	0-4
Set 9	10.0	7332.	0-12

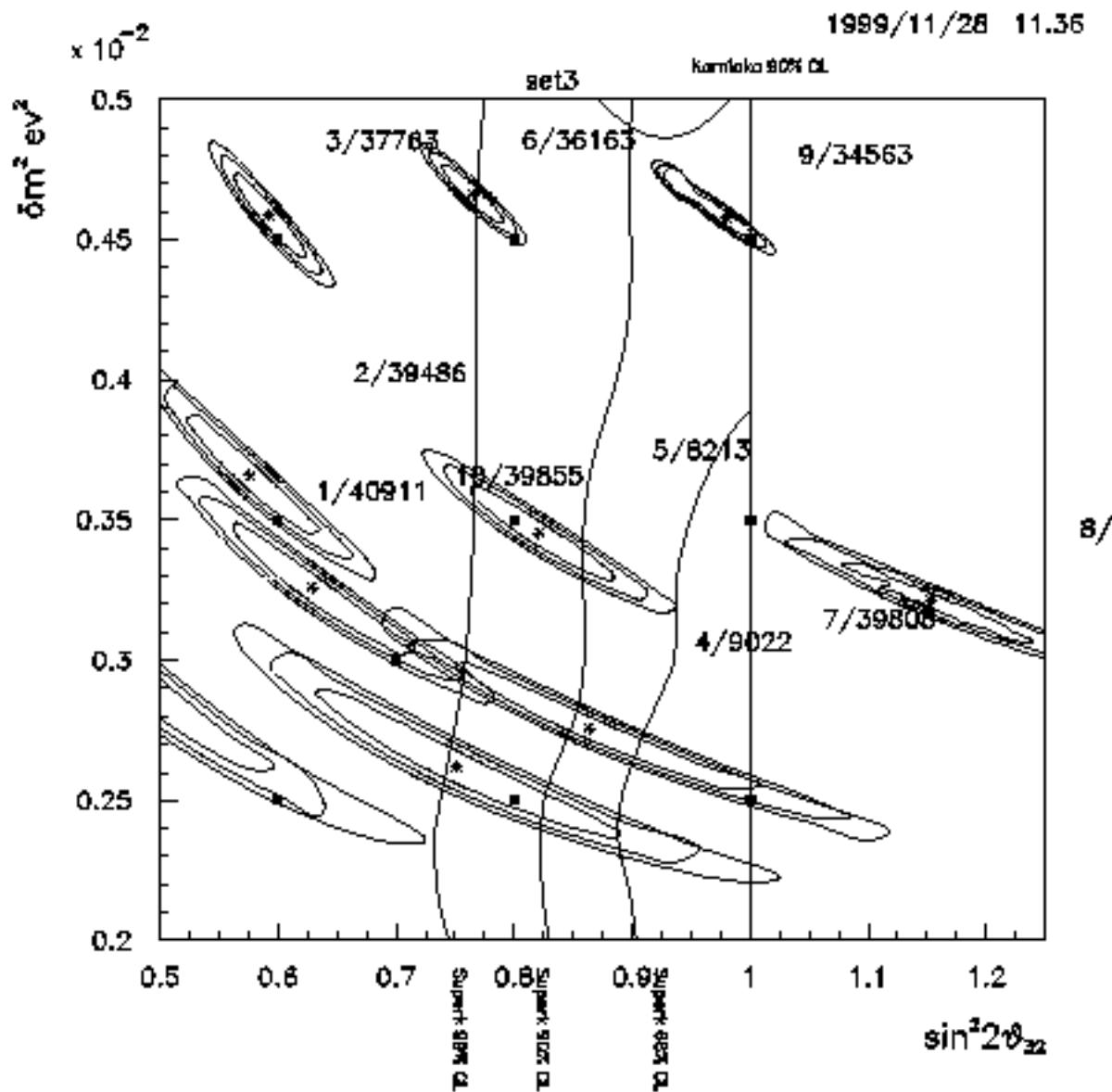
Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $\nu_\mu \rightarrow \nu_\mu$

- 10 GeV μ^- 732 km baseline fit 0-4 GeV



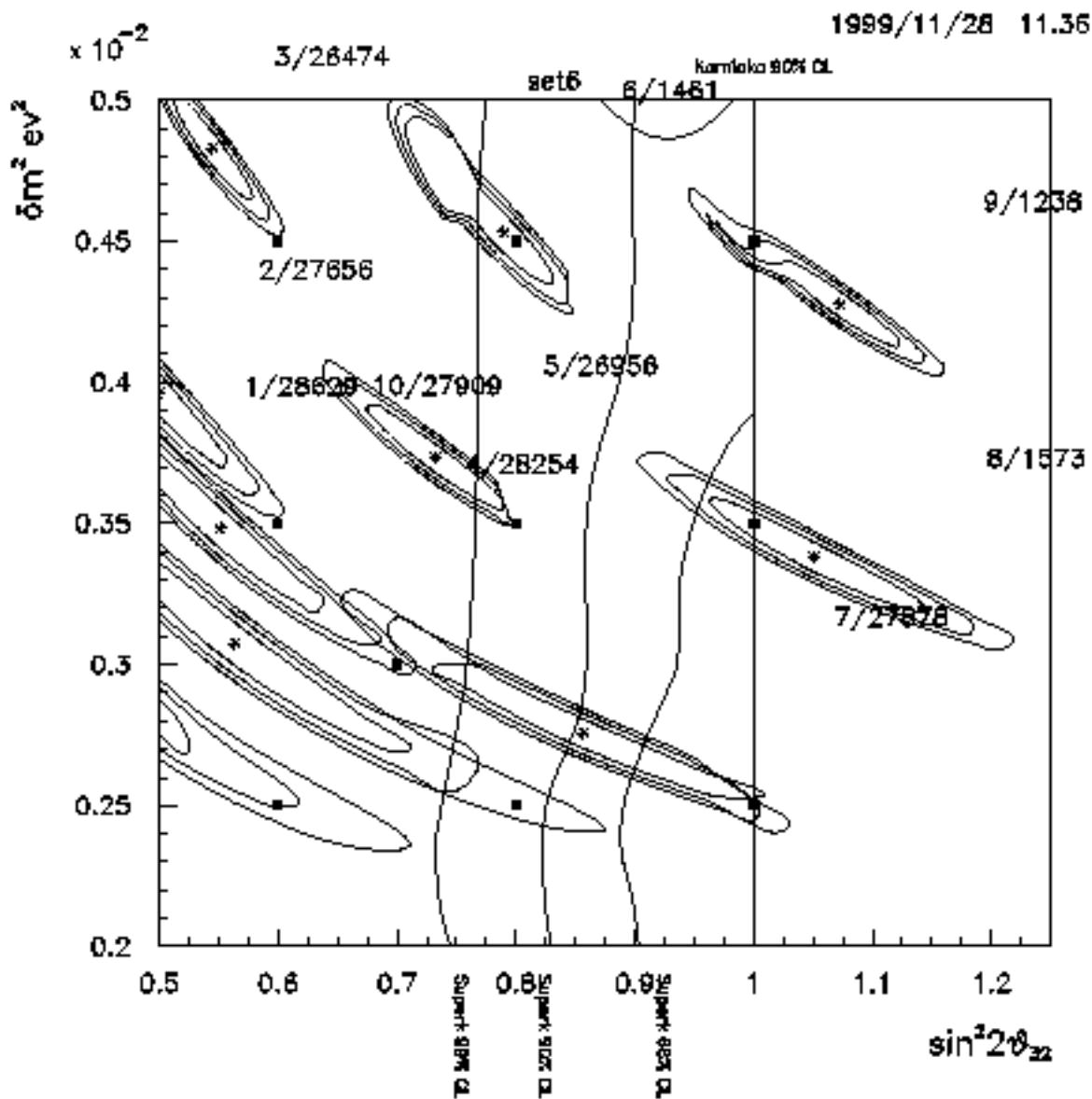
Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $\nu_\mu \rightarrow \nu_\mu$

- 30 GeV μ^- 732 km baseline fit 0-12 GeV



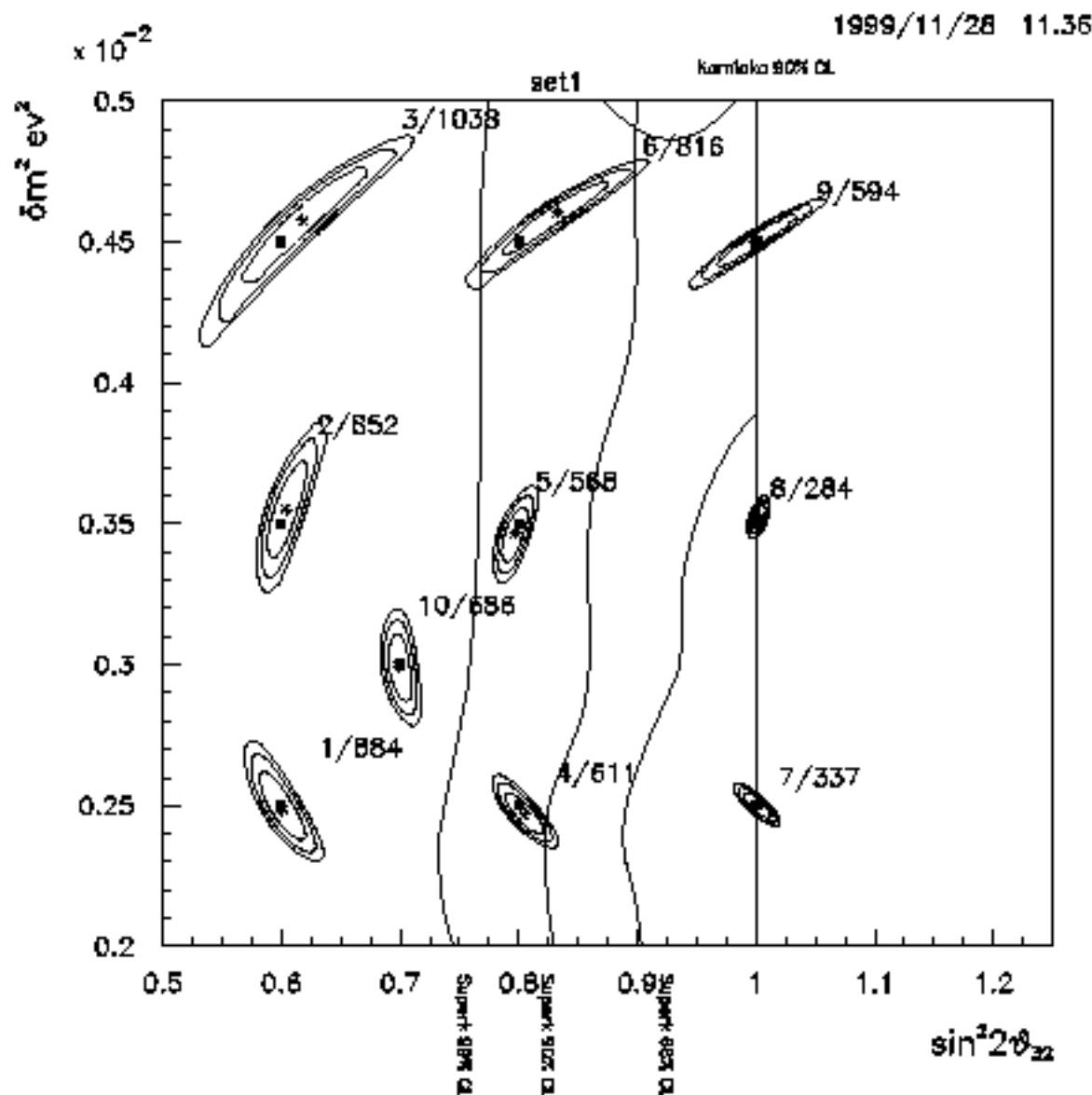
Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $\nu_\mu \rightarrow \nu_\mu$

- 50 GeV μ^- 732 km baseline fit 0-12 GeV



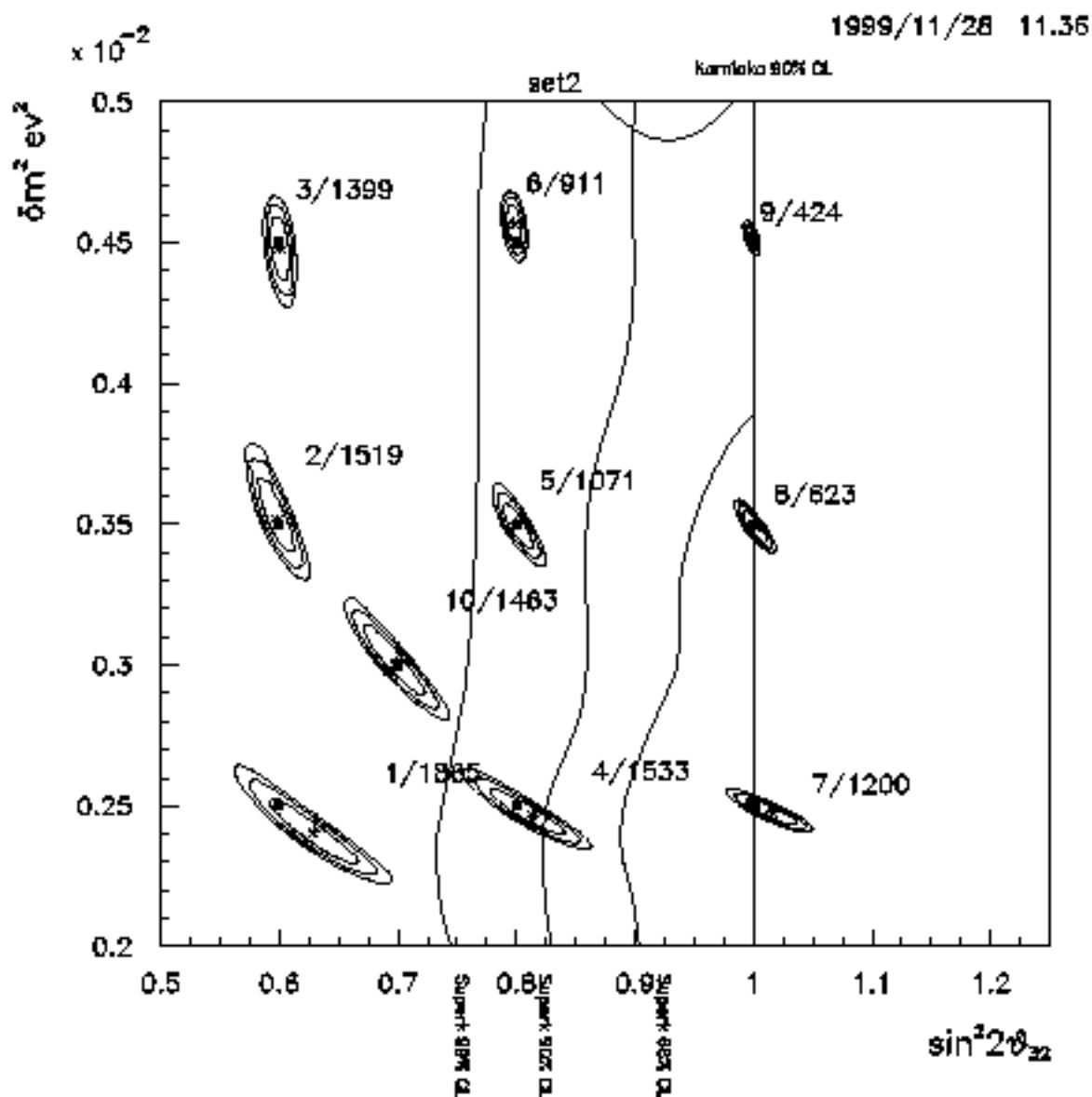
Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $\nu_\mu \rightarrow \nu_\mu$

- 10 GeV μ^- 2800 km baseline fit 0-10 GeV



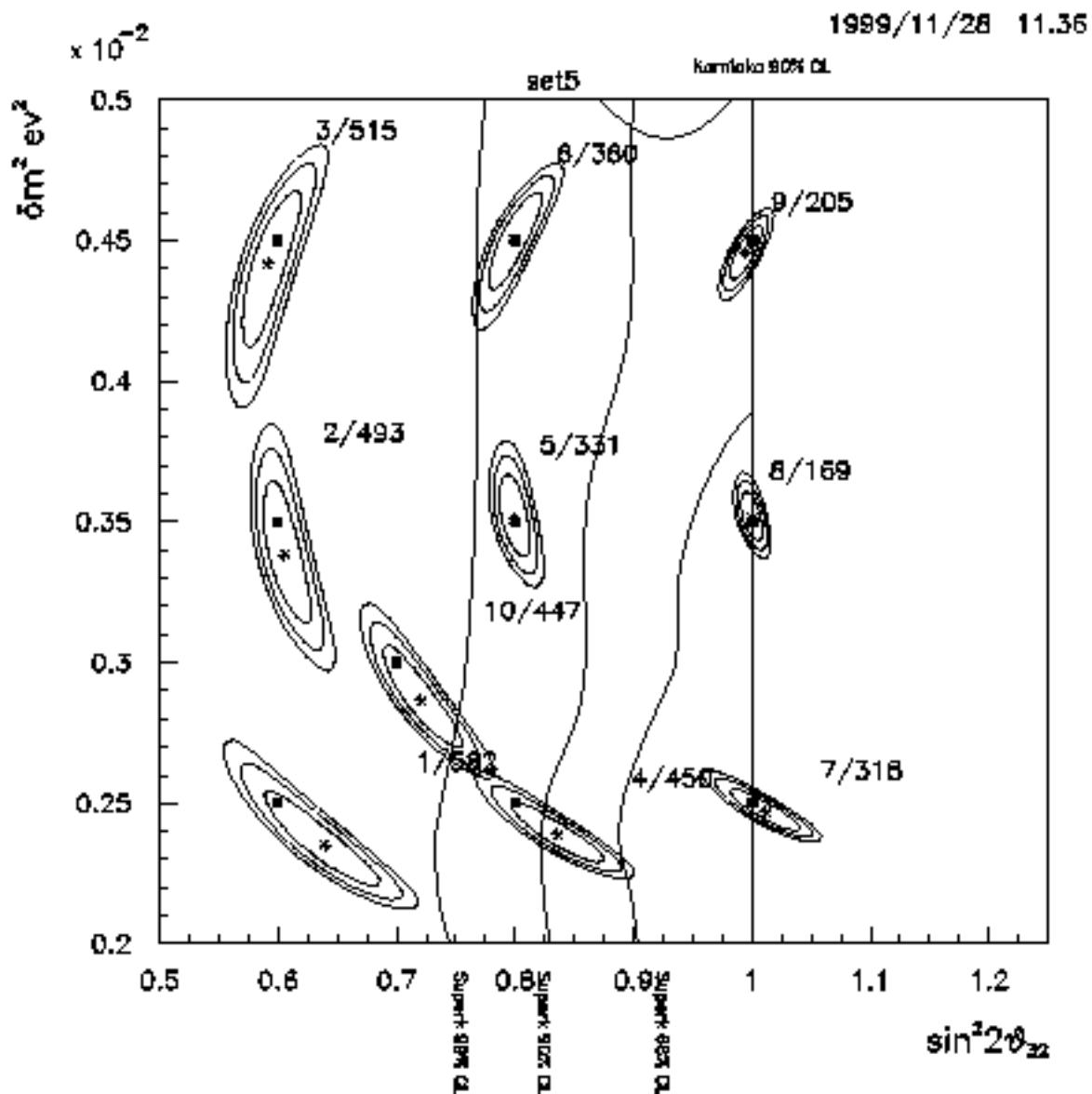
Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $\nu_\mu \rightarrow \nu_\mu$

- 30 GeV μ^- 2800 km baseline fit 0-12 GeV



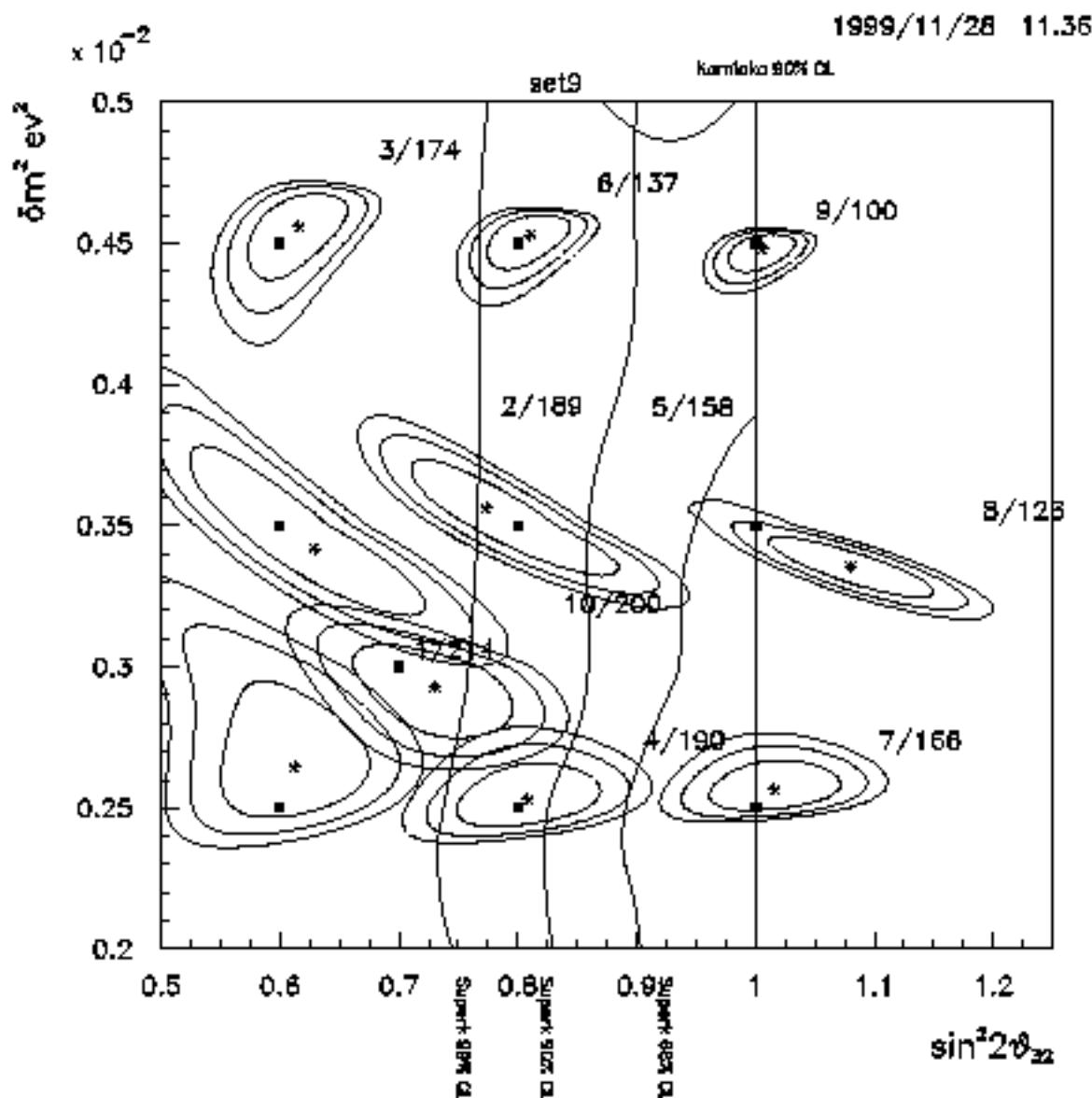
Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $\nu_\mu \rightarrow \nu_\mu$

- 50 GeV μ^- 2800 km baseline fit 0-10 GeV



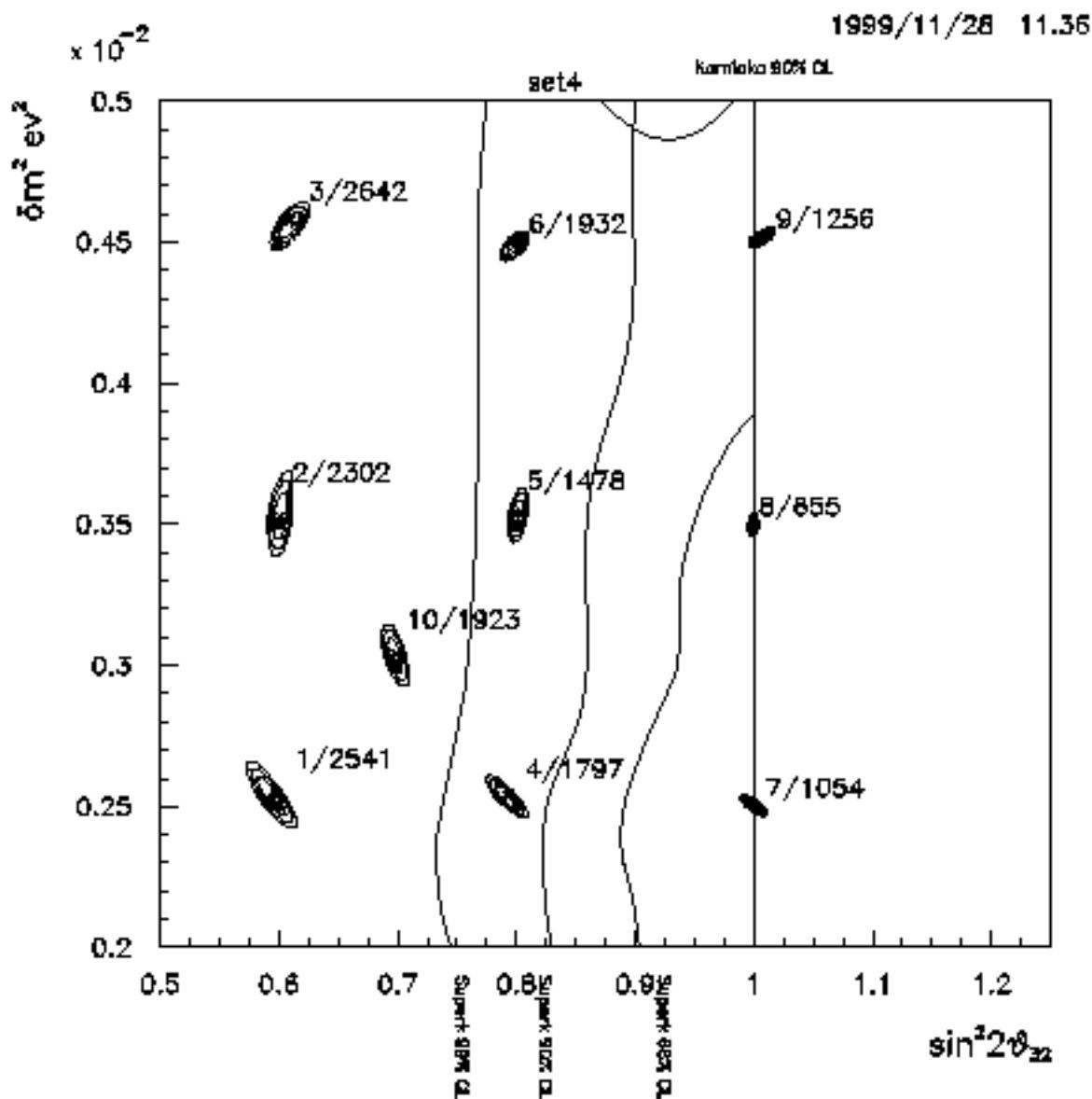
Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $\nu_\mu \rightarrow \nu_\mu$

- 10 GeV μ^- 7332 km baseline fit 0-12 GeV



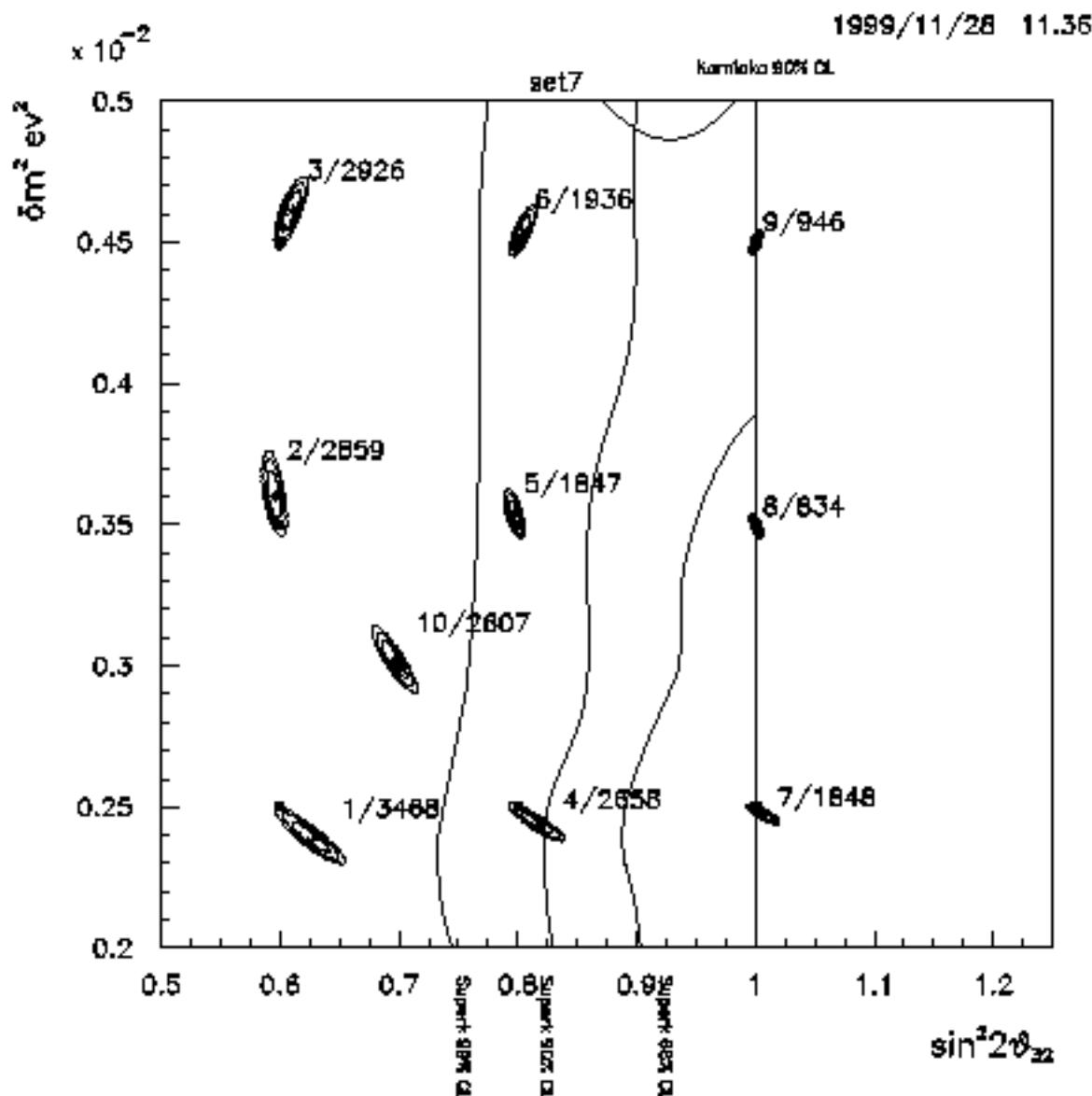
Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $\nu_\mu \rightarrow \nu_\mu$

- 30 GeV μ^- 7332 km baseline fit 0-25 GeV



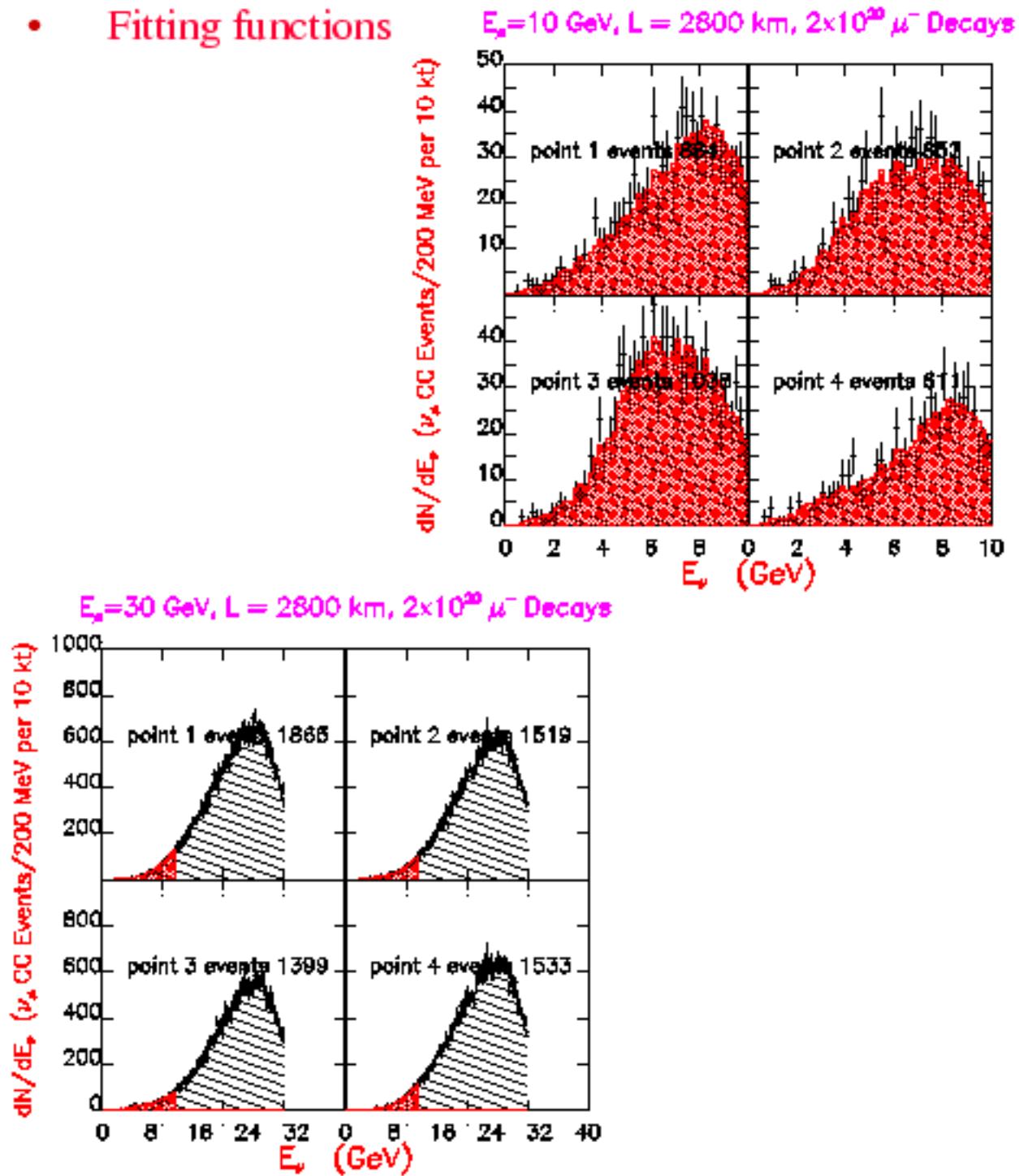
Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $\nu_\mu \rightarrow \nu_\mu$

- 50 GeV μ^- 7332 km baseline fit 0-28 GeV



Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $\nu_\mu \rightarrow \nu_\mu$

- Fitting functions

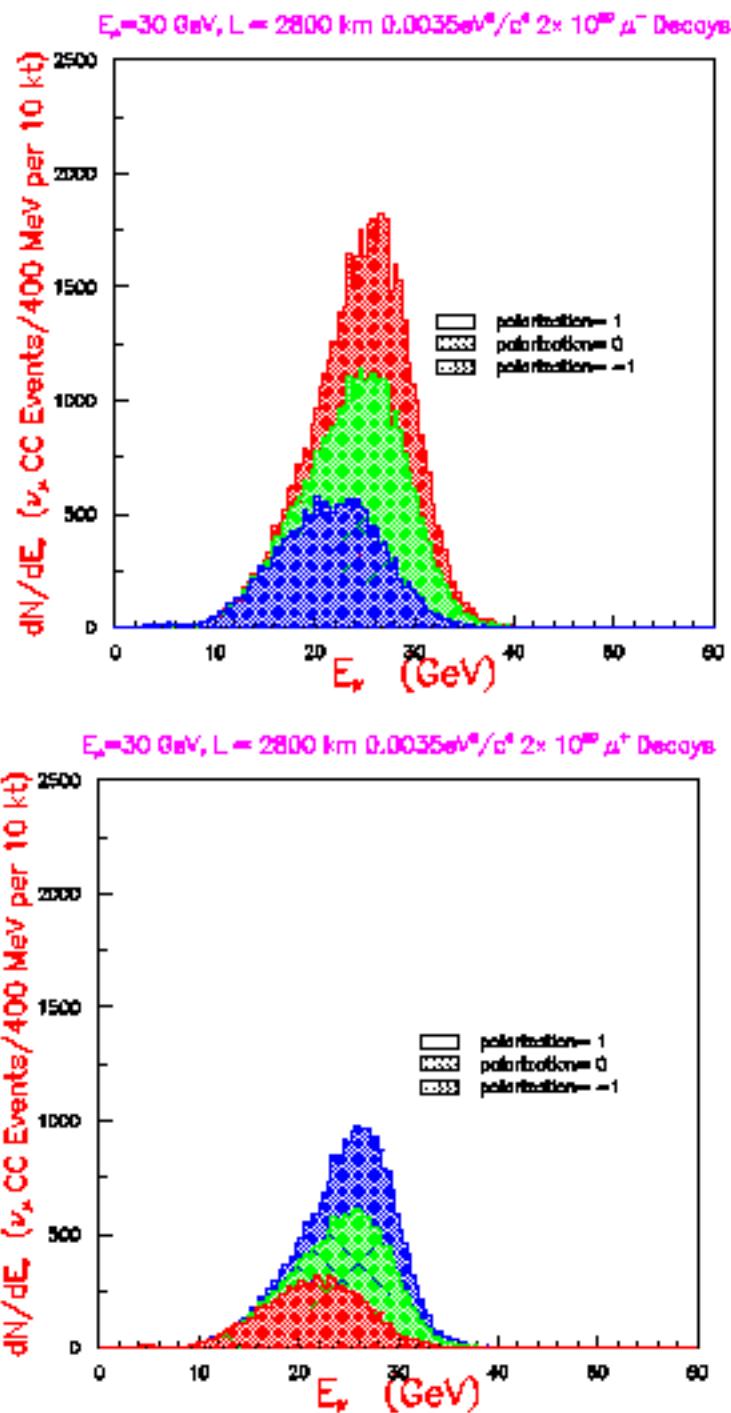


Fitting for δm^2 and $\sin^2 2\theta$ for the disappearance channel $\nu_\mu \rightarrow \nu_\mu$

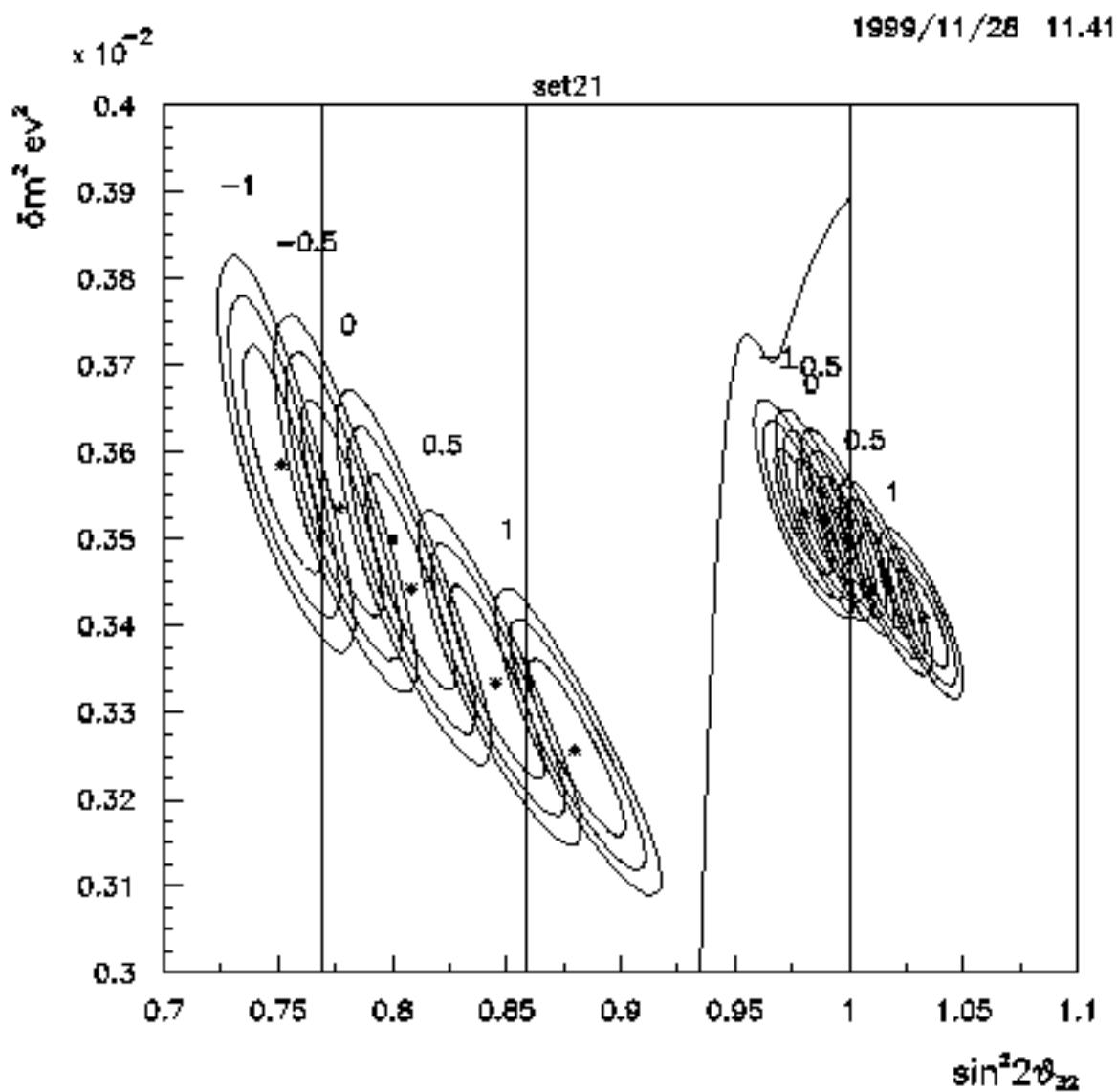
Summary of errors for $\sin^2 2\theta = 1.0$, $\delta m^2 = 0.35 \times 10^{-2} \text{ eV}^2/\text{c}^4$.

Baseline length km	Muon energy GeV	Error $\sin^2 2\theta$	Error δm^2
732	10	7.6%	6.7%
732	30	14%	8.9%
732	50	17%	12%
2800	10	1.1%	2.4%
2800	30	2.0%	3.2%
2800	50	1.8%	4.9%
7332	10	1.3%	6.3%
7332	30	0.57%	1.2%
7332	50	0.64%	1.4%

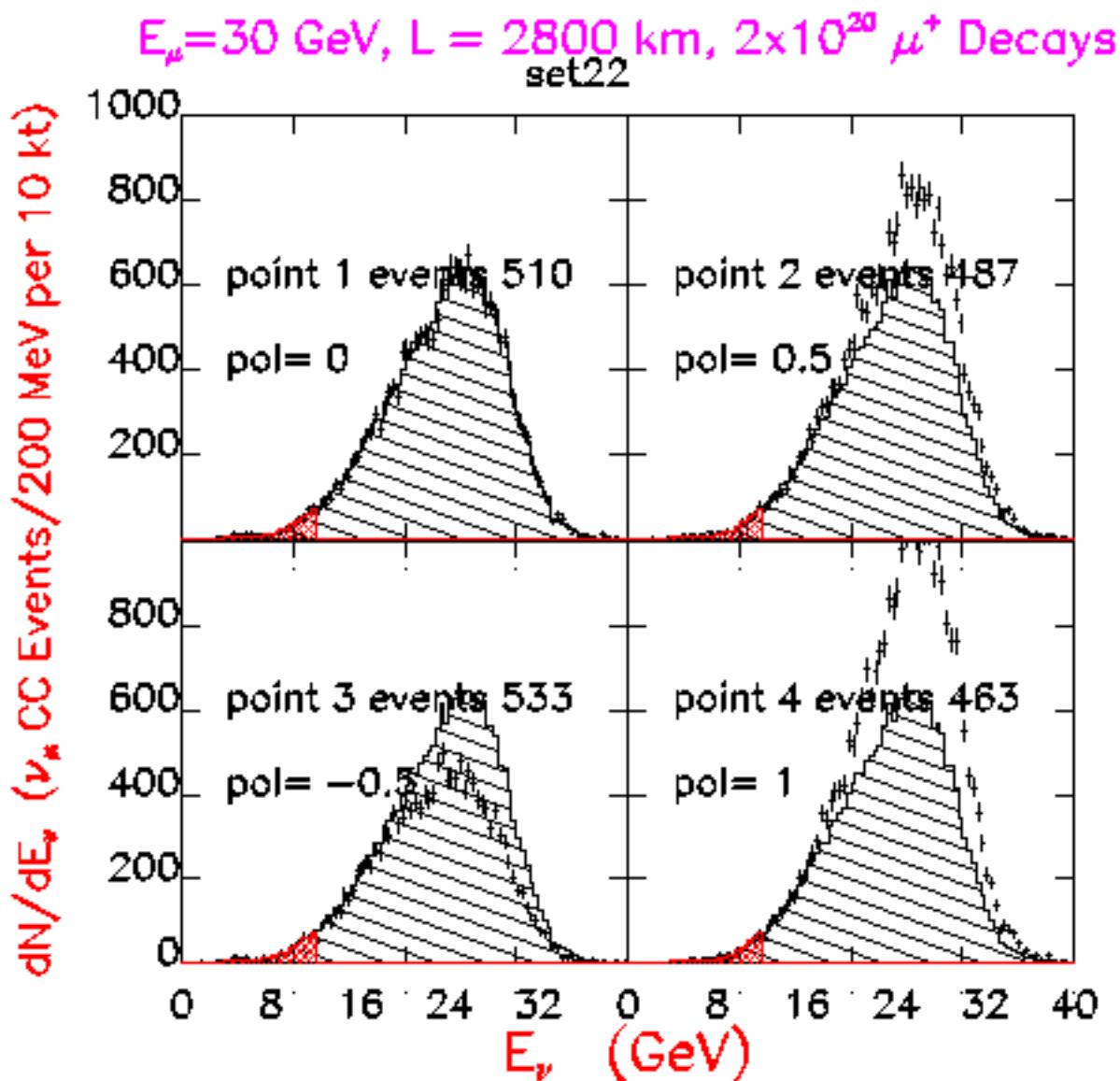
Distributions with Polarization



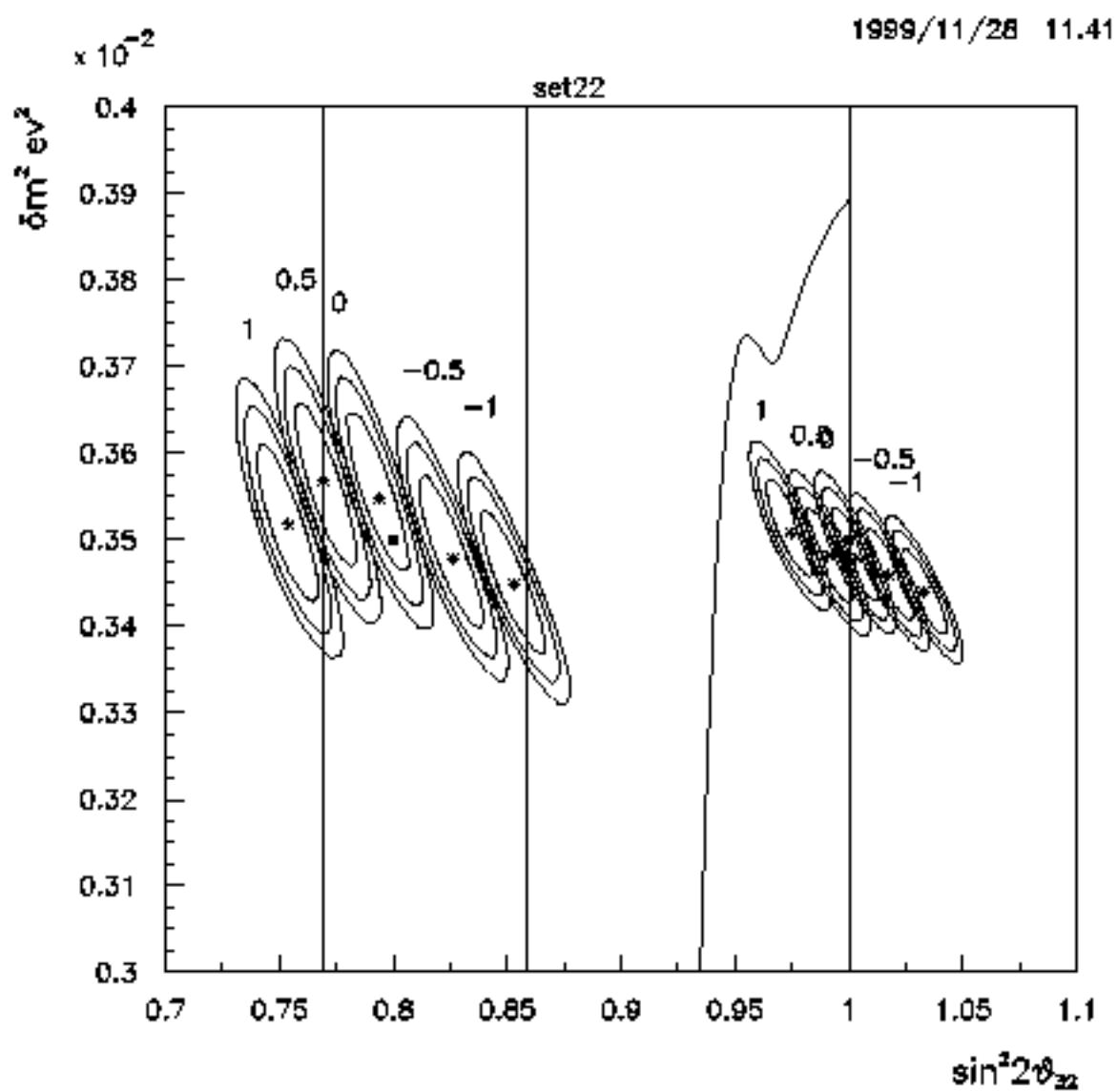
Polarized μ^+ decay fits



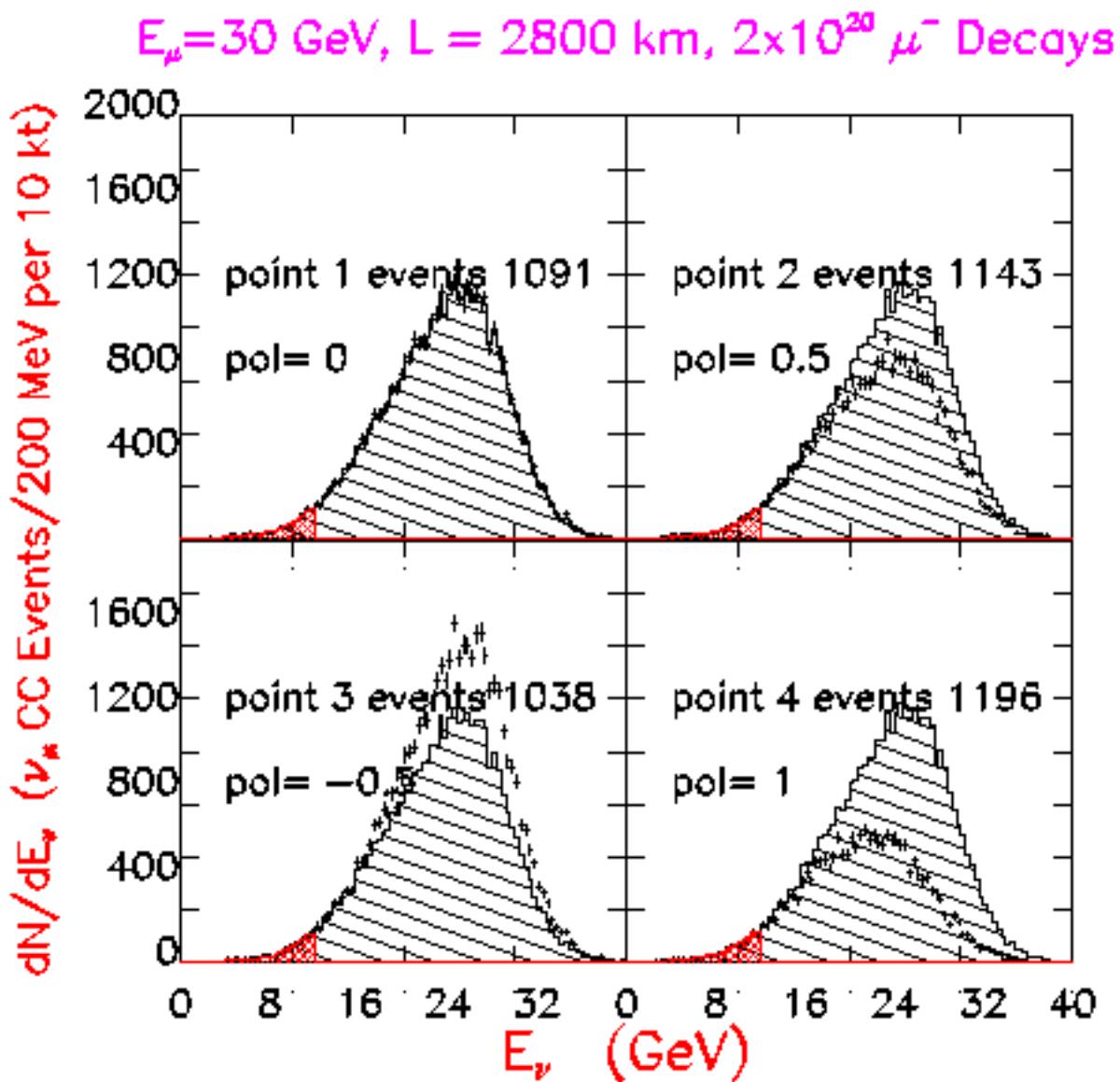
Polarized μ^+ decay fits



Polarized μ decay fits



Polarized μ decay fits



Conclusions

- Polarization must be reckoned with.
- Ideally one should design a ring in which polarization precesses and is preserved. It is measured by measuring the decay electrons in the using a calorimeter around the beam pipe in the storage ring. Using this, one can measure the muon enrgy precisely and the neutrino and anti-neutrino spectra can be predicted turn to turn. This may turn out to be a powerful tool for discerning the parentage of oscillated events.
- Failing this, a bow-tie option may be considered, where the polarization is preserved and is the same from turn to turn. It should then be measured using a near detector.
- Muon storage rings offer unprecedented precision in studying the lepton flavor sector.